THE GALVANIC CIRCUIT INVESTIGATED MATHEMATICALLY

BY

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BERLIN, 1827

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WITH A PREFACE BY THE EDITOR

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EDITOR'S PREFACE.

A SUFFICIENT reason for republishing an English translation of the wonderful book of Professor G. S. Ohm is the difficulty with which the only previous translation (that of Taylor's Scientific Memoirs) is procurable.

Besides this, however, the intrinsic value of the book is so great, that it should be read by all electricians who care for more than superficial knowledge.

It is most remarkable to note at this time, how completely Ohm has investigated the subject, and how far in advance of his age he was.

It is well said by Chrystal, in his great Encyclopædia Britannica article on Electricity, that "Ohm rendered a great service to the science of electricity by publishing his mathematical theory of the galvanic circuit." Before his time, the quantitative circumstances of the electric current had been indicated, in a vague way, by the use of the terms "intensity" and "quantity," to which no accurately defined meaning was attached. Ohm's service consisted in introducing and defining the accurate notions — electromotive force, current strength, and resistance. He indicated the connection of these with experiment, and stated his famous law that the electromotive force divided by the resistance is equal to the strength of the current.

It is perhaps worth recalling, that Henry Cavendish, in his secret and solitary researches, made experiments in 1781, the results of which practically anticipated Ohm's considerations; but Cavendish having satisfied himself, did not apparently consider it worth while to take any one else into account.

In Maxwell's "Introduction to the Cavendish Papers," we find it stated that, "One of the most important investigations which Cavendish made was to find, as he expressed it, 'what power of the velocity the resistance is proportional to.'" (See *Cavendish Researches*, Arts. 574, 575, 629, 686.)

Cavendish means by "resistance" the whole force which resists the current, and by "velocity" the strength of the current through unit of area of the section of the conductor.

By four different series of experiments on the same solution in wide and in narrow tubes, Cavendish found that the resistance (in his sense) varied as the

1.08, 1.03, 0.976, and 1.00

power of the velocity.

This is the same as saying that the resistance (in the modern sense) varies as the

0.08, 0.03, — 0.024

power of the strength of the current in the first three sets of experiments, and in the fourth set, that it does not vary at all.

This result, obtained by Cavendish in January, 1781, is an anticipation of the law of electric resistance discovered independently by Ohm, and published in 1827. It was not until long after the latter date that the importance of Ohm's law was fully appreciated, and that the measurement of electric resistance became a recognized branch of research.

The exactness of the proportionality between the electromotive force and the current in the same conductor seems, however, to have been admitted, rather because nothing else could account for the consistency of the measurements of resistance obtained by different methods, than on the evidence of any direct experiments.

Some doubts having been suggested with respect to the mathematical accuracy of Ohm's law, the subject was taken up by the British Association in 1874, and the experiments of Professor Chrystal, by which the exactness of the law, as it relates to metallic conductors, was tested by currents of every degree of intensity, are contained in the report of the British Association for 1876.

The laws of the strength of currents in multiple and divided circuits are accurately stated by Cavendish in Arts. 417, 597, 598.

I have found also that Cavendish communicated to the Royal Society in 1775, "An account of some attempts to imitate the effects of the Torpedo by electricity," in which he delivers the following remarkable statement:—

"When a jar is electrified, and any number of different circuits are made between its positive and negative side, some electricity will necessarily pass along each; but a greater quantity will necessarily pass along those in which it meets with less resistance than those in which it meets with more;" and also "Some electricians indeed seem to have supposed that the electric fluid passes only along the shortest and readiest circuit; but, beside that, such a supposition would be contrary to what is observed in all other fluids—it does not agree with experience."

The above is not referred to as any depreciation of the work of Ohm, but simply as an interesting point in connection therewith.

Ohm's work stands alone, and, reading it at the present time, one is filled with wonder at his prescience, respeet for his patience and prophetic soul, and admiration at the immensity and variety of ground covered by his little book, which is indeed his best monument.

It is my experience that many quote Ohm's law, and talk of Ohm's law, who know little or nothing of Ohm himself, or of his book; and it seems to me that a large proportion of these, will welcome the opportunity to read, in their own language, what Ohm himself has to say about his own law, which so long has practically been sealed up, and has been accessible only at second hand.

With the hope that this opportunity will be utilized fully, and that this classic may become a familiar friend, the "Mathematical Consideration of the Galvanic Circuit" is once more launched.

T. D. L.

MELROSE, MASS., 1891.

AUTHOR'S PREFACE.

I HEREWITH present to the public a theory of galvanic electricity, as a special part of electrical science in general, and shall successively, as time, inclination, and means permit, arrange more such portions together into a whole, if this first essay shall in some degree repay the sacrifices it has cost me. The circumstances in which I have hitherto been placed have not been adapted either to encourage me in the pursuit of novelties, or to enable me to become acquainted with works relating to the same department of literature throughout its whole extent. I have therefore chosen for my first attempt a portion in which I have the least to apprehend competition.

May the well-disposed reader receive the performance with the same love for the object as that with which it is sent forth!

THE AUTHOR.

BERLIN, May 1, 1827.

THE GALVANIC CIRCUIT INVESTIGATED MATHEMATICALLY.

INTRODUCTORY.

THE design of this Memoir is to deduce strictly from a few principles, obtained chiefly by experiment, the *rationale* of those electrical phenomena which are produced by the mutual contact of two or more bodies, and which have been termed galvanic; its aim is attained if by means of it the variety of facts be presented as unity to the mind. To begin with the most simple investigations, I have confined myself at the outset to those cases where the electricity developed propagates itself in one dimension only.

They form, as it were, a scaffolding to a greater structure, and contain precisely that portion, the more accurate knowledge of which may be gained from the elements of natural philosophy, and which also by reason of its greater accessibility may be given in a more strict form. To achieve this especial purpose, and at the same time as an introduction to the subject itself, I give as a forerunner of the compressed mathematical investigation, a more free, but not on that account a less connected, general view of the process and its results.

Memoir based on Three Laws. — Three laws, of which the first expresses the mode of distribution of the electricity within one and the same body; the second, the mode of dispersion of the electricity in the surrounding atmosphere; and the third, the mode of appearance of the electricity at the place of contact of two heterogeneous bodies, forms the basis of the entire Memoir, and at the same time contains everything that does not lay claim to being completely established.

The two latter are purely experimental laws; but the first from its nature is, in part at least, theoretical.

With regard to this first law, I have started from the supposition that the communication of the electricity from one particle takes place directly only to the one next to it, so that no immediate transition from that particle to any other situate at a greater distance occurs. The magnitude of the transition between two adjacent particles under otherwise exactly similar circumstances, I have assumed as being proportional to the difference of their temperatures.

With respect to the dispersion of electricity in the atmosphere, I have retained the law deduced from experiments by Coulomb, according to which, the loss of electricity, in a body surrounded by air, in a given time, is in proportion to the force of the electricity, and to a co-efficient dependent on the nature of the atmosphere.

A simple comparison of the circumstances under which Coulomb performed his experiments, with those at present known respecting the propagation of electricity, showed, however, that in galvanic phenomena, the influence of the atmosphere may generally be disregarded. In Coulomb's experiments, for instance, the electricity driven to the surface of the body was engaged in its entire expanse in the process of dispersion in the atmosphere; while in the galvanic circuit, the electricity almost constantly passes through the interior of the bodies, and consequently but a very small part can enter into mutual action with the air; so that the dispersion can comparatively be only very inconsiderable. This consequence, deduced from the nature of the circumstances, is confirmed by experiments; in it lies the reason why the second law seldom comes into consideration.

The mode in which electricity makes its appearance at the place of contact of two different bodies, or the electrical tension of these bodies, I have thus expressed: when dissimilar bodies touch one another, they constantly maintain at the point of contact the same difference of *potential*.[1]

With the help of these three fundamental positions, the conditions to which the propagation of electricity in bodies of any

[1] **Ohm's word was "Spannung."**—***Electroscopic Force.*** **Ed.**

kind and form is subjected may be stated. The form and treatment of the differential equations thus obtained are so similar to those given for the propagation of heat by Fourier and Poisson, that even if there existed no other reasons, we might with perfect justice draw the conclusion that there exists an intimate connection between these natural phenomena; and this similarity increases as we continue to pursue the subject.

["Ohm, misled by the analogy between electricity and heat, entertained an opinion that a body when raised to a high potential becomes electrified throughout its substance, as if electricity were compressed into it, and was thus by means of an erroneous opinion led to employ the equations of Fourier to express the true laws of conduction of electricity through a long wire, long before the real reason of the appropriateness of these equations had been suspected." — *Electricity and Magnetism.* MAXWELL. 1881. Vol. i. p. 422.]

These mathematical researches are of the most difficult order, and can therefore obtain general acquiescence but gradually; it is thus a fortunate chance that in an important part of the propagation or transmission of electricity in consequence of its peculiar nature, those difficulties almost entirely disappear.

To place this portion before the public is the object of the present Memoir, and therefore so many only of the complex cases have been admitted as seemed requisite to render the transition apparent.

The nature and form commonly given to galvanic apparatus favor the propagation of the electricity only in one dimension; and the velocity of its diffusion combined with the constancy with which a source of galvanic electricity acts, is the cause of the assumption by the galvanic phenomena, for the most part, of a character which does not vary with time. These two conditions, to which galvanic phenomena are most frequently subjected, viz., change of the electric state in a single dimension, and its independency of time, are however precisely the reasons why the investigation is brought to a degree of simplicity, which is not surpassed in any branch of natural philosophy, and is altogether adapted to secure incontrovertibly to mathematics the possession of a new field of physics, from which it had hitherto remained almost totally excluded.

Fluid Portions of a Circuit introduce Chemical Changes which complicate the Consideration. — The chemical changes which so frequently occur in some — generally fluid — portions of a galvanic circuit, deprive the result of much of its natural simplicity, and by the complications they produce, conceal to a considerable extent the peculiar progression of the phenomenon; they cause an unexampled change of the phenomenon which gives rise to many apparent exceptions to the rule, frequently amounting even to contradictions, in so far as the sense of this word is itself not in contradiction to nature.

[De La Rive writes thirty years later: "There is a special resistance due to the mere fact of the passage of the current from a solid into a liquid, or from a liquid into a solid. We call this resistance the *resistance to passage.* It always takes place while a liquid is being traversed by an electric current; for in order to put this liquid into the circuit, solid conductors called electrodes must necessarily be employed. This resistance to passage . . . is the result of the electro-chemical phenomena which in virtue of the decomposing action of the current necessarily occur on the surfaces of the solid conductors that are in contact with the liquids in which they are transmitting this current." — *Treatise on Electricity.* De La Rive. 1856. Vol. ii. p. 73.]

I have distinctly separated the consideration of such galvanic circuits in which no portion undergoes a chemical change from those whose activity is disturbed by chemical action, and have devoted separate space in the Appendix to the latter.

This total separation of the two parts constituting the whole, and as might be thought the less dignified position of the latter, will find in the following circumstance a sufficient explanation.

A theory which asserts itself to be an enduring and fruitful one, must in all its consequences be in accordance with observation and experiment.

It seems to me that with respect to the first of the above-mentioned parts, this is sufficiently established, partly by the previous experiments of others, and partly by some performed by myself, which first made me acquainted with the theory here developed, and subsequently rendered me entirely devoted to it. Such is not the case with regard to the second part. A more accurate experimental verification is in this case still required, for which I have neither time nor means; and therefore it is for the present set in a corner, from whence, if it prove to be worth the trouble, it may be hereafter withdrawn, and may then also be further matured under better nursing.

First case —

A Circuit of Uniform Material and Uniform Size of Conductor.—By means of the first and third fundamental positions, we obtain a distinct insight into the primary galvanic phenomenon, as follows. Imagine, for instance, a ring everywhere of equal thickness and homogeneous, having at any one place one and the same electrical potential; i.e., inequality in the electrical state of two surfaces situated close to each other; from which causes, when they have come into action, and the equilibrium is consequently disturbed, the electricity will in its endeavor to re-establish itself, if its mobility be solely confined to the extent of the ring, flow off on both sides.

If this tension were merely momentary, the equilibrium would very soon be re-established; but if the tension is permanent, the equilibrium can never be restored; but the electricity by virtue of its expansive force, which is not sensibly restrained, produces in an almost inappreciably brief space of time, a state which approximates closely to equilibrium, and consists in this: that by the constant transmission of the electricity, a perceptible change in the electric condition of the parts of the conductor through which the current passes is nowhere produced.

The peculiarity of this state, which occurs also frequently in the transmission of light and heat, arises from the fact that each particle of the conducting medium situated in the circuit of action receives each moment just the same amount of the transmitted electricity from the one side as it gives off to the other, and therefore constantly retains an unchanged quantity.

Now, since by reason of the first fundamental position the electrical transition only takes place directly from the one particle to the other, and is under otherwise similar circumstances, determined according to its energy by the electrical difference of the two particles, this state must evidently indicate itself on the ring uniformly excited in its entire thickness, and similarly constituted in all its parts, by a constant change in the electric condition, originating from the point of excitation, proceeding uniformly through the whole ring, and finally again returning to the place of excitation; whilst at this place itself, a sudden spring in the electric condition, constituting the tension, is, as was previously stated, constantly perceptible. In this simple separation or division of the electricity lies the key to the most varied phenomena.

The mode of separation of the electricity has been completely determined by the preceding observation; but the absolute force of the electricity at the various parts of the ring still remains uncertain. This property may best be conceived by imagining the ring, its nature remaining otherwise unaltered, opened at the point of excitation and extended in a straight line, and representing the force of the electricity at such point by the length of a perpendicular line erected upon it; that directed upwards may represent a positive, but that downwards, a negative electrical state of the point.

Potential or Tension and its Fall may be Graphically Represented.—The line A B

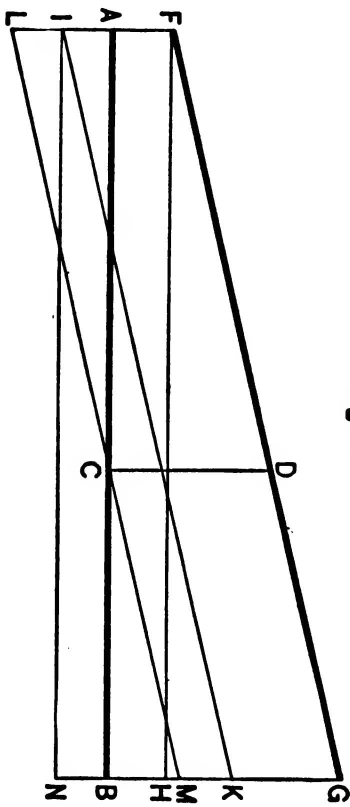

Fig. 1

(Fig. 1) may accordingly represent the ring extended in a straight line, and the lines A F and B G perpendicular to A B may indicate by their lengths the force of the positive electricities situated at the extremities A and B.

If now the straight line F G be drawn from F to G, also F H parallel to A B, the position of F G will give the mode of separation of the electricity,[1] and the quantities B G,—A F or G H the tension occurring at the extremities of the ring; and the force of the electricity at any other place C, may easily be expressed by the length of C D drawn through C perpendicularly to A B.

But from the nature of the voltaic excitation, the absolute magnitudes of the lines A F and B G are not determined, but merely the amount of the tension, or the length of the line G H; consequently the mode of separation may be represented quite as well by any other line parallel to the former; *e.g.*, by I K, for which the tension still constantly retains the same value expressed by K N, because the ordinates

[1] Or, as we should now express it, "the difference of potential existing between the extremities or the total electromotive force at work in the circuit."— Ed.

situated at present below A B assume a relation opposed to their former one.

Which of the infinitely numerous lines parallel to F G would express the actual state of the ring, cannot be generally stated, but must in each case be separately determined from the circumstances which occur. Moreover, it is easily conceived that as the position of the line sought is given, it would be completely determined for one single part of the ring by the determination of any one of its points, or, in other words, by the knowledge of the electric force.

If, for instance, the ring lost all its electricity at the place C, by abduction,[1] the line L M drawn through C parallel to F G would in this case express with perfect certainty the electrical state of the ring.

It is to this variability in the separation of the electricity that the changeableness of the phenomenon peculiar to the voltaic circuit is to be attributed.

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[1] If the point C of the ring, that is to say, were reduced to zero potential by connecting it with a conducting body such as the earth, whose potential is assumed to be zero.

It may further be added, that it is evidently immaterial whether the position of the line F G with respect to that of A B be fixed; or, whether the position of the line F G remain constantly the same, and the position of A B with respect to it, be altered. The latter course is by far the simpler where the separation of the electricity assumes a more complex form.

Second case:—

A Circuit composed of any Number of Sections of Varying Size and Material.—The conclusions reached, which hold for a ring homogeneous throughout its whole extent, may easily be extended to a ring composed of any number of heterogeneous parts, if each part be of itself homogeneous, and of the same thickness.

We may here take as an example of this extension a ring composed of two heterogeneous parts. Let this ring be imagined as before, open at one of its places of excitation and stretched out to form the right line A B C (Fig. 2), so that A B and B C indicate the two heterogeneous portions of the ring.

The perpendiculars A F and B G, will represent by their lengths the electrical

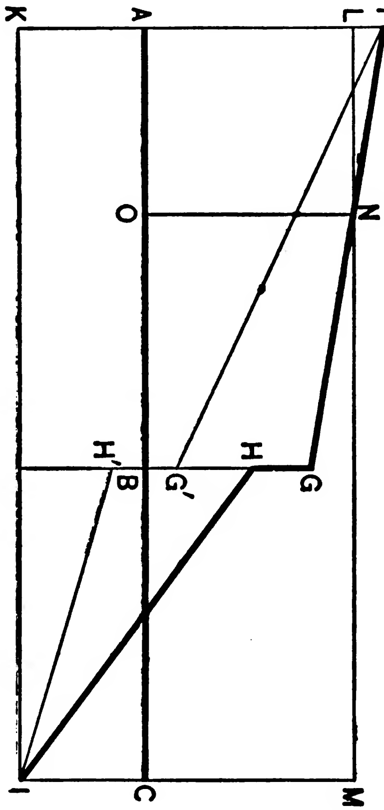

Fig. 2

forces present at the extremities of the part A B; on the other hand, B H and C I, those present at the extremities of the part B C; accordingly, A F+C I or F K will represent the tension at the opened place of excitation, and G H the tension occurring at B at the point of contact. Now, if we only bear in mind the permanent state of the circuit, the straight lines F G and H I will, from the reasons above mentioned, indicate by their position the mode of separation of the electricity in the ring; but whether the line A C will keep its place, or must be advanced further up or down, remains uncertain, and can only be found out in each distinct case by other separate considerations.

If, for instance, the point O of the circuit is[1] touched abductively, and thus deprived of all electricity, O N would disappear; and therefore the line L M drawn through N parallel with A C would, in this case, give the position of A C required. It is hence evident how sometimes this, sometimes another, position of the line A C in the figure F G H I, representing the separation of the electricity, may be the one suited to the circumstances; and here-

[1] Or brought to zero potential. — Ed.

in we recognize the source of the variability of galvanic phenomena already mentioned.

It is however essential, in order that we may be able to thoroughly appreciate the present case, to attend to a circumstance which has purposely been hitherto left unmentioned, in order that the various considerations might be separated as distinctly as possible. The distances F K and G H are indeed given by the tensions existing at the two places of excitation, but the figure F G H I is not yet wholly determined by this alone.

For instance, the points G and H might move down towards G′ and H′, so that G′ and H′ would equal G H, giving rise to the figure F G′ H′ I, which would indicate quite a different mode of separation of the electricity, although the individual tensions in it still retain their former magnitude.

Consequently, if that which has been stated with respect to the circuit of two members is to require a sense no longer subject to any arbitrary explanation, this uncertainty must be removed.

The first fundamental law affects this in the following way: For since the state of the ring alone, independent of the time, is regarded, each section must, as has already been stated, receive in every moment the same quantity of electricity from one side as it gives off to the other. This condition occasions upon such portions of the ring as have fully the same constitution at their various points, the constant and uniform change in the separation which is represented in the first figure by the straight line F G, and in the second by the straight lines F G and H I. But when the geometrical or the physical nature of the ring changes in passing from one of its component parts to another, the reason of this constancy and uniformity is no longer in force; consequently the manner in which the several straight lines are combined into a complete figure must first be deduced from other considerations. To facilitate the object, I will separately consider the geometrical and physical difference of the single parts, each independently.

A Circuit composed throughout of the Same Material but of Varying Size.—Let us first suppose that every section of the part B C is m times smaller than in the part A B, while both parts are composed of the same substance; the electric state of the ring which is independent of time, and which requires that everywhere throughout the ring just as much electricity be received on one side as is given off from the other, can evidently only exist under the condition that the electric transition from one particle to the other in the same time within the portion B C is m times greater than in the portion A B; because it is only in this manner that the action in both parts can maintain equilibrium.

But in order to produce this m times greater transition of the electricity from particle to particle, the electrical difference between particle and particle within the portion B C must, according to the first fundamental position, be m times greater than in the portion A B; or when this determination is transferred to the figure, the line H I must fall *m* times more on equal portions, or have an *m* times greater "dip" than the line F. G. By the expression "dip" (Gefälle) is to be understood the difference of such ordinates as belong to two places distant one unit of length from each other.

From this consideration results the following rule:[1] *The dips of the lines F G and H I in the portions A B and B C, composed of like substance, will be inversely to each other as the areas of the sections of these parts.* By this the figure F G H I is now fully determined.

A Circuit Conductor formed of Two Parts of like Cross-section but of Different Material.—When the parts A B and B C of the ring have the same section but are composed of different substances, the transition of the electricity will then no longer be dependent solely on the progressive

[1] That is to say, the steepness or extent of the fall of potential in equal lengths of different portions of the circuit of like material but of different size, will be directly proportional to the resistance of such portions.—Ed.

change of electricity in each part from particle to particle, but also at the same time on the peculiar nature [specific conductivity] of each substance.

This differing mode of distribution of the electricity, caused solely by the material nature of the bodies, whether it have its origin in the peculiar structure or in any other peculiar relation of the bodies to electricity, establishes a distinction in the electrical conductivities of the various bodies; and even the present case may afford some information respecting the actual existence of such a distinction, and give rise to its more accurate determination.

In fact, since the ring composed of the two parts A B and B C differs from the homogeneous one only in this respect, that the two parts are formed of two different substances, a difference in the dip of the two lines F G and H I will make known a difference in the conductivity of the two substances, and one may serve to determine the other. In this way we arrive at the following position, supplying the place of a definition: *In a ring consisting of two parts, A B and B C, of like sections but formed of different substances, the dips of the lines F G and H I are inversely as the conducting powers of the two parts.*[1]

If we have once ascertained the conducting powers of the various substances, they may be employed to determine the dips of the lines F G and H I, in every case that may occur.

By this, then, the figure F G H I is entirely determined.

The determination of the conductivity from the separation of the electricity is rendered very difficult from the low intensity of galvanic electricity, and from the imperfection of the requisite apparatus; subsequently we shall obtain an easier means of effecting this purpose.

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[1] That is, in a circuit conductor formed of two portions of like size but different material, the fall of potential in the two portions respectively is directly proportional to their specific resistances. — Ed.

A Circuit Conductor formed of Two Parts which are neither of the Same Size or of the Same Material. — From these two particular cases we may now ascend in the usual way to the general one where the two prismatic parts of the ring neither possess the same section, nor are constituted of the same substance.

In this case the dips of the two parts must be in the inverse ratio of the products of the sections and powers of conduction.

We are hereby enabled to determine completely the figure F G H I in every case, and also to distinguish perfectly the mode of electrical separation in the ring.

Generalizations deducible from the Foregoing.—All the peculiarities, hitherto considered separately, of the ring composed of two heterogeneous parts, may be summed up in the following manner: *In a galvanic circuit consisting of two heterogeneous prismatic parts, there takes place in regard to its electrical state a sudden transition from the one part to the other at each point of excitation, forming the ten-*

sion there occurring, and from one extremity of each point to the other a gradual and uniform transition ; and the dips of these two transitions are inversely proportional to the products of the conductivities and sections of each part.

Proceeding in this manner, we are able without much difficulty to inquire into the electrical state of a ring composed of three or more heterogeneous parts, and to arrive at the following general law: *In a galvanic circuit consisting of any indefinite number of prismatic parts, there takes place in regard to its electrical state at each place of excitation a sudden transition, from one part to the other, forming the tension there prevailing, and within each part a gradual and uniform transition from the one extremity to the other ; and the dips of the various transitions are inversely proportional to the products of the conductivities and sections of each part.* From this law may easily be deduced the entire figure of the separation for each particular case, as I will now show by an example.

Let A B C D (Fig. 3) be a ring composed of three heterogeneous parts, open at one

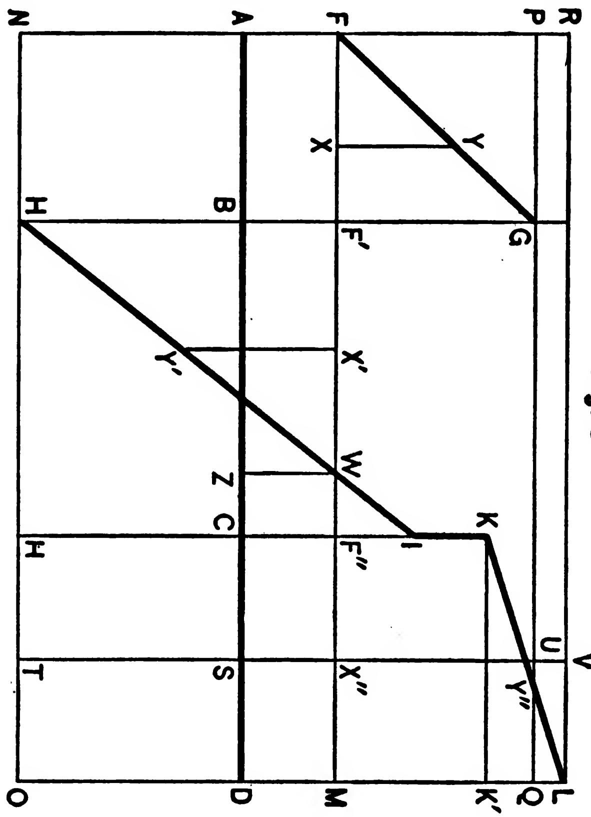

Fig. 3

of its places of excitation, and extended in a straight line. The straight lines F G, H I, K L, represent by their position the mode of separation of the electricity in each individual part of the ring, and the lines A F, B G, B H, C I, C K, and D L, drawn through A, B, C and D perpendicular to A D, such quantities that G H, K I, and L M or D L — A F show by their length the magnitude of the tensions occurring at the individual places of excitation. From the known magnitude of these tensions, and from the given nature of the single parts A B, B C, and C D, the figure of the electrical separation has to be entirely determined.

If we draw straight lines parallel to A D, through the points F, H and K, meeting the line drawn through B, C and D perpendicular to A D, in the points F′, H′, K′, then according to what has already been demonstrated, the lines G F′, I H′, and L K′ are directly proportional to the lengths of the parts A B, B C, and C D, and inversely proportional to the products of the conductivity and section of the same part; consequently the relations of the lines GF′, I H′, and L K′ to each other are given. Further, that G F′ + I H′ + L K′ = G H — K I

+ (D L — A F = L M) is also known, as the tensions represented by G H, K I and D L — A F are given. From the given relations of the lines G F′, I H′, L K′ and their known sum, these lines may now be found individually; the figure F G H I K L is evidently then entirely determined. But the position of this figure with respect to the line A D remains from its very nature still undecided.

If we recollect that proceeding in the same direction A D, the tensions represented by G H, and D L — A F or L M, indicate a sudden sinking of the electric force at the respective places of excitation, that represented by I K on the contrary a sudden rise of the force; and that tensions of the first kind are regarded and treated as positive quantities, while tensions of the latter kind are considered as negative quantities, we find the above example leads us to the following generally valid rule: *If we divide the sum of all the tensions of the ring composed of several parts into the same number of portions which are directly proportional to the lengths of the parts, and* *inversely proportional to the products of their conductivities and their sections, these portions will give in succession the amount of gradation which must be assigned to the straight lines belonging to the single parts and representing the separation of the electricity; at the same time the positive sum of all the tensions indicates a general rise, and on the contrary the negative sum of all the tensions a general depression of those lines.*

Determination of Potential at any Point of a Circuit. — We will now proceed to the determination of the electric force at any given position in every galvanic circuit, and here again we lay down Fig. 3 as a basis.

For this purpose let a a' a'' indicate the tensions existing at B, C, and between A and D, so that in this case also a and a'' represent additive, a' on the contrary, a subtractive line, and λ, λ', λ'', any lines which are directly as the lengths of the parts A B, B C, and C D, and inversely as the products of the conductivities and sections of the same parts; further, let $a + a' + a'' =$ A, and $\lambda + \lambda' + \lambda'' =$ L, then, according to the law just ascertained.

G F′ is a fourth proportional to L, A and λ,

I H′ a fourth proportional to L A and λ',

L K′ a fourth proportional to L, A and λ''.

Draw the line F M through F parallel to A D, regard this line as the axis of the abscissæ, and erect at any given points X, X′, X″, the ordinates X Y, X′ Y′, X″ Y″, we obtain their respective values, thus: —

In the first place we have, since A B = F F″

A B : G F′ = F X : X Y, whence follows:

$$X\,Y = \frac{F\,X \,.\, G\,F'}{A\,B},$$ or if we substitute for G F′ its value $\frac{A\,.\,\lambda}{L}$

$$X\,Y = \frac{A}{L} \cdot \frac{F\,X \,.\, \lambda}{A\,B}.$$

If now x represent a line such that A B : F X = λ : x, then $X\,Y = \frac{A}{L} \cdot x$.

Secondly, since B C and F′ X′ are equal to the lines drawn through I and Y′ to G H parallel to A D,
B C : I H′ = F′ X′ : F′ H − X′ Y′, whence

$$- X' Y' = \frac{I H' . F' X'}{B C} - F' H$$

or, since F′ H = G H − G F′

$$- X' Y' = \frac{I H' . F' X'}{B C} + G F' - a.$$

If now for I H′ and G F′ we substitute their values $\frac{A : \lambda''}{L}$, $\frac{A . \lambda'}{L}$ and $\frac{A . \lambda}{L}$, we obtain

$$- X' Y' = \frac{A}{L} \left(\lambda + \frac{F' X' . \lambda'}{B C} \right) - a;$$

and if by x' we represent a line such that B C : F′ X′ = $\lambda' : x'$, then

$$- X' Y' = \frac{A}{L} (\lambda + x') - a.$$

Thirdly, since C D = K K′ and F″ X″ is equal to the part of K K′ which extends from K to the line X″ Y″, we have C D : L K′ = F″ X″ : X″ Y″ − K F″, whence

$$X'' Y'' = \frac{L K' . F'' X''}{C D} + K F'',$$

or since $K F'' = K I + I H' - F' H$ and $F'H = G H - G F'$,

$$X'' Y'' = \frac{L K' . F'' X''}{C D} + I H' + G F' - (a + a').$$

If now for $L K'$, $I H'$, $G F'$, we substitute their values $\frac{A . \lambda''}{L}$, $\frac{A . \lambda'}{L}$, $\frac{A . \lambda}{L}$, we obtain

$$X'' Y'' = \frac{A}{L} \left(\lambda + \lambda' + \frac{F'' X'' . \lambda''}{C D} \right) - (a + a');$$

and if by x'' we represent a line such that $C D : F'' X'' = \lambda'' : x''$ we have

$$X'' Y'' = \frac{A}{L} (\lambda + \lambda' + x'') - a + a'.$$

These values of the ordinates, belonging to the three distinct parts of the circuit and different in form from each other, may be reduced as follows to a common expression.

For if F is taken as the origin of the abscissæ, F X will be the abscissa corresponding to the ordinate X Y, which belongs to the homogeneous part A B of the ring, and x will represent the length corresponding to this abscissa in the reduced proportion of A B : λ. In like manner F X′ is the abscissa corresponding to the ordinate X′ Y′ which is composed of the parts F F′ and F′ X′ belonging to the homogeneous portions of the ring, and λ x' are the lengths reduced in the proportions of A B : λ and B C : λ' corresponding to these parts. Lastly, F X″ is the abscissa corresponding to the ordinate X″ Y″ which is composed of the parts F F′, F′ F″, F′ X″, belonging to the homogeneous portions of the ring, and λ, λ', x'' are the lengths reduced in the proportions of A B : λ, B C : λ', C D : λ''.

If in consequence of this consideration we call the values x, $\lambda + x'$, $\lambda + \lambda'$ x'' *reduced abscissæ* and represent them generally by y, we obtain

$$\mathrm{X\,Y} = \frac{\mathrm{A}}{\mathrm{L}} \cdot y$$

$$-\mathrm{X'\,Y'} = \frac{\mathrm{A}}{\mathrm{L}} \cdot y - a$$

$$\mathrm{X''\,Y''} = \frac{\mathrm{A}}{\mathrm{L}} \cdot y - (a + a'),$$

and it is evident that L is the same in reference to the whole length A D or F M as y is to the lengths F X, F X′, F X″, on account of which L is termed the entire reduced length of the circuit. Further, if we consider that for the abscissa corresponding to the ordinate X Y the tension has experienced no abrupt change, but that for the abscissa corresponding to the ordinate X′ Y′ the tension has experienced the abrupt changes a, a'; and if we represent generally by O the sum of all the abrupt changes of the tensions for the abscissa corresponding to the ordinate y, then all the values found for the various ordinates are contained in the following expression : —

$$\frac{A}{L} \cdot y - O.$$

But these ordinates express, when an arbitrary constant, corresponding to the length A F, is added to them, the electric forces existing at the various parts of the ring. If therefore we represent the electric force at any place generally by u, we obtain the following equation for its determination: —

$$u = \frac{A}{L} y - O + c,$$

in which c represents an arbitrary constant.

This equation is generally true, and may be thus expressed in words: *The force of the electricity at any point of a galvanic circuit composed of several parts, is ascertained by finding the fourth proportional to the reduced length of the entire circuit, the reduced length of the part belonging to the abscissa, and the sum of all the tensions, and by increasing or diminishing the difference between this quantity and the sum of all the abrupt changes of tension for the given abscissa by an arbitrary quantity which is constant for all parts of the circuit.*

Current is of Equal Strength in all Parts of the Circuit. — When the determination of the electric force at each point of the circuit has been effected, it only remains to determine the magnitude of the electric current. Now, in a galvanic circuit of the kind hitherto mentioned, the quantity of electricity passing through a section of it in a given time is everywhere the same, because at all places and in each moment the same quantity in the section leaves it on the one side as enters it from the other, but in different circuits this quantity may be very different; therefore, in order to compare the actions of several galvanic circuits *inter se*, it is requisite to have an accurate determination of this quantity, by which the magnitude of the current in the circuit is measured. This determination may be deduced from Fig. 3 in the following manner: It has already been shown that the force of the electric transition in each instant from one element to the adjacent one is given by the electric difference between the two existing at that time, and by a magnitude dependent upon the kind and form of the particles of the body, viz., the conductivity of the body. But the electrical difference of the elements in the part B C, for instance, reduced to a constant unit of distance, will be expressed by the dip of the line

H I or by the quotient $\frac{\text{I H}'}{\text{B C}}$; if, therefore, we now indicate by χ the magnitude of the conductivity of the part B C,

$$\frac{\chi \,.\, \text{I H}'}{\text{B C}}$$

will express the force of the transition from element to element, or the strength of the current in the part B C; consequently, if ω represent the magnitude of the section in the part B C, the quantity of electricity passing in each instant from one section to the adjacent one, or the *magnitude* of the current will be expressed by

$$\frac{\chi \,.\, \omega \,.\, \text{I H}'}{\text{B C}};$$

or if S represent this magnitude of the current, we have

$$\text{S} = \frac{\chi \,.\, \omega \,.\, \text{I H}'}{\text{B C}},$$

and if we substitute for I H′ its value

$$\frac{\text{A} \,.\, \lambda'}{\text{L}},$$

$$\text{S} = \frac{\text{A}}{\text{L}} \cdot \frac{\chi \,.\, \omega \,.\, \lambda'}{\text{B C}}.$$

Laws of the Current. — Hitherto the λ, λ', λ'' have represented lines which are proportional to the quotients formed of the lengths of the parts A B, B C, C D, and the products of their conductivities and their sections.

If we restrict for the present this determination, which leaves the absolute magnitude of the lines λ, λ', λ'' uncertain, so that the magnitudes λ λ', λ'' shall not be merely proportional to the said quotients, but shall likewise be equal to them, and henceforth vary this limitation in accordance with the meaning of the expression "reduced lengths," the first of the two preceding equations becomes

$$S = \frac{I\,H'}{\lambda'},$$

which gives the following generally: *The magnitude of the current in any homogeneous portion of the circuit is equal to the quotient of the difference between the electrical forces present at the extremities of snch portion divided by its reduced length.*

This expression for the forces of the

current will continue to be subsequently employed.

The second of the former equations passes, by the adopted change, into

$S = \frac{A}{L}$, which is generally true, and already reveals the equality of the force of the current at all parts of the circuit; in words it may be expressed thus:—

The force of the current in a galvanic circuit is directly as the sum of all the tensions, and inversely as the entire reduced length of the circuit, bearing in mind that at present by reduced length is understood the sum of all the quotients obtained by dividing the actual lengths corresponding to the homogeneous parts by the product of the corresponding conductivities and sections.

[“*Reduced Length.*”—The length of a copper wire of a given thickness, the resistance of which is equivalent to the sum of the resistance in a circuit, Ohm calls a reduced length.”—*Bakerian Lecture*, June 15, 1843, WHEATSTONE.

[The last of the two above expressions is in substance Ohm's Law, and is its first distinct formulation.

Wheatstone, in his Bakerian Lecture, 1843, discussed processes for determining the Constant of a Voltaic Circuit, and described the laws of the current, processes of electrical measurement, and instruments for making such measurements; he remarks that the said "instruments and processes are all founded on the principles established by Ohm in his theory of the voltaic circuit," which theory he terms "beautiful and comprehensive."

"Viewing the law of the electric circuit," he continues, "from the point at which the labors of Ohm has placed us, there is scarcely any branch of experimental science in which so many and such various phenomena are expressed by formulæ of such simplicity and generality." The statement of Ohm's Law made in this lecture is as follows: "Let F denote the force of the current, E the electromotive forces, and R the resistances, then $F = \frac{E}{R}$."]

["The absolute intensity of the electricity, that travels in the form of a current through a closed circuit, depends upon two circumstances alone, — the force or forces that produce the electricity, and which we may call electromotive forces, and the resistances to conductibility presented by all the circuit taken together. This latter element, which had never previously been taken into account, was pointed out by myself, both in 1825, in the Memoir to which I have already alluded above" (Ann. de Ch. et de Phys. tome xxviii. p. 190), "and also in the subsequent researches that I published in 1828 and afterwards. In an important work which appeared in 1827, M. Ohm, as a result of purely theoretical speculations, came to the conclusion that the force of the current in a closed circuit is directly proportional to the sum of the electromotive forces that are in activity in the circuit, and which we will call E, and inversely proportional to the total resistances of all parts of the circuit, which we will designate by R; in other words, that the intensity of the current, I, is equal to the sum of the electromotive forces, divided by the sum of the resistances: $I = \frac{E}{R}$." — *Treatise on Electricity.* De La Rive. Vol. ii. pp. 77, 78.]

From the equation determining the force of the current in a galvanic circuit, in conjunction with the one previously found, by which the electric force at each point of the circuit is given, may be deduced with ease and certainty all the phenomena belonging to the voltaic circuit. The former had already been some time since derived from exhaustive experiments[1] with an apparatus which allows of an accuracy and certainty of measurement not suspected in this department; the latter expresses all the observations pertaining to it, which already exist in great

[1] Schweigger's Jahrbuch, 1826. Part 2.

number, with the most exact fidelity, which also continues where the equation leads to results no longer comprised in the circle of previously published experiments. Both advance uninterruptedly hand in hand with nature, as we now hope to demonstrate by a short statement of their consequences; at the same time, it seems necessary to observe, that both equations refer to all possible galvanic circuits[1] *whose state is permanent*, consequently they comprise the voltaic combination as a particular case, so that the theory of the pile needs no separate comment.

In order to be distinct, I shall constantly, instead of employing the equation

$$u = \frac{A}{L} y - O + c,$$

only take the third figure, and therefore will merely remark here, once for all, that all the consequences drawn from it hold generally.

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[1] Ohm, it appears, thus clearly recognizes the applicability of his law to "circuits whose state is permanent" only. — Ed.

In the next place, the circumstance that the separation of the electricity, diffusing itself over the galvanic circuit, maintains at the different places a permanent and unchangeable gradation, although the force of the electricity is variable at one and the same place, deserves a closer inspection. This is the reason of that magic mutability of the phenomena which enables us to determine at pleasure the action of a given place of the galvanic circuit on the electrometer, and enables us to produce it instantly.

To explain this peculiarity I will return to Fig. 3. Since the figure of separation, F G, H I, K L, is always wholly determined from the nature of any circuit, although its position with respect to the circuit A D, as was seen, is fixed by no inherent cause, but can assume any change produced by a movement common to all its points in the direction of the ordinates, the electrical condition of each point of the circuit expressed by the mutual position of the two lines may be varied constantly, and at will, by external influences. When, for example, A D is at any time the position representing the actual state of the circuit, so that, therefore, the ordinate S Y″ expresses by its length the force of the electricity at that point of the circuit to which such ordinate belongs, then the electrical force corresponding to the point A, at the same time will be represented by the line A F. If now the point A be brought in some way to zero potential, the line A D will be brought into the position F M and the force previously existing in the point S will be expressed by the length X″ Y″; this force, therefore, has suddenly undergone a change, corresponding to the length S X″.

The same change would have occurred if the circuit had been brought to zero potential at the point Z, because the ordinate Z W is equal to that of A F.

If the circuit were so treated at the point where the two parts A B and B C join but so that the contact was made within the part B C, we should have to imagine A D advanced to N O; the electrical force at the point S would in this case be increased to the force indicated by T Y″.

But if the zero producing contact took place still at the same point; viz., where the parts A B and B C touch each other, but within the part A B, the line A D would be moved to P Q, and the force belonging to the point S would sink to the negative force expressed by U Y″. If, lastly, the battery had been arranged so that it had been brought to zero at the point D, we should have prescribed for the line A D the position R L, and the electrical force at the point S would have assumed the negative force indicated by V Y″.

The law of these changes is obvious, and may be expressed generally thus: *Each point of a galvanic circuit undergoes mediately, in regard to its outwardly acting electrical force, the same change which is produced immediately at any other point of the circuit by external influences.*

Since each point of a voltaic circuit undergoes, of itself, the same change to which a single point is brought, the change in the amount of electricity, extending over the whole circuit, is proportional, on the one hand, to the sum of all the points; i. e., to the extent over which the electricity is diffused in the circuit, and, moreover, to the change in the electric force produced at one of these places.

From this simple law the following distinct phenomena result. If we call r the space over which the electricity is diffused in the galvanic circuit, and imagine this circuit touched at any single point by a non-conducting body, and designate by u_1 the electric force at this point before contact, by u that after contact, the change produced in the force at this point is $u_1 - u$; consequently the change of the whole quantity of electricity in the circuit is $(u_1 - u)\, r$.

If now, we suppose that the electricity in the touched body is diffused over the space R, and is at all points of equal strength, and at the same time, that at the place of contact itself, the circuit and the body possess the same electric force; viz., u, it is evident u R will be the quantity of electricity imparted to the body, and $(u_1 - u)\, r = u\, \mathrm{R}$, whence we obtain

$$u = \frac{u_1\, r}{r + \mathrm{R}}.$$

The intensity of the electricity received by the body will, therefore, be the more nearly equal to that which the circuit possessed at the place of contact before being touched, the smaller R *is with respect to r; it will amount to the half when* R $= r$, *and become weaker, as* R *becomes greater in comparison with r.*

Since these changes are merely dependent on the relative magnitude of the spaces r and R, and not in any sense or degree on the qualitative nature of the circuit, they are determined not only by the actual dimensions of the circuit, but also even by foreign masses brought into conducting connection with the circuit. If we connect this fact with the theory of the condenser, we arrive at an explanation of all the relations of the voltaic circuit to the condenser, noticed by [1]Jäger, which is very surprising.

[1] Gilbert's Annalen, vol. xiii.

I content myself with regard to this point, to refer to the Memoir itself, to give room here for the insertion of some new peculiarities of the voltaic circuit. The mode of separation of the electricity, within a homogeneous part of the circuit, is determined by the magnitudes of the dips of the lines F G, H I, K L (Fig. 3), and there again by the magnitudes of the ratios

$$\frac{G.F'}{A\ B}, \frac{I\ H'}{B\ C}, \frac{L K'}{C\ D}.$$

But, as was already shown,

$$G.\ F' = \frac{A}{L} \cdot \lambda, \ I\ H' = \frac{A}{L} \cdot \lambda',$$

$$L\ K' = \frac{A}{L} \cdot \lambda'';$$

hence it may be seen, without much trouble, that the magnitude of the dip of the line corresponding to any part of the circuit, and representing the separation of the electricity, is obtained by multiplying the value $\frac{A}{L}$ by the ratio of the reduced to the actual length of the same part. If, therefore, (λ) represented the reduced length of any homogeneous part of the circuit and (l) its actual length, the magnitude of the dip of the straight line, belonging to this part, and representing the separation of the electricity is

$$\frac{A}{L} \cdot \frac{(\lambda)}{(l)},$$

which expression, if we designate by (χ) the conductivity, and by (ω) the section of the same part may also be written thus: —

$$\frac{A}{L} \cdot \frac{(\lambda)}{(\chi)\,(\omega)}$$

This expression leads to a more detailed knowledge of the separation of the electricity in a voltaic circuit. For since A and L designate values which remain identical for each part of the same circuit, it is evident *that the dips in the separate homogeneous parts of a circuit are to one another inversely as the products of the conductivity, and the section of the part.*

If, consequently, a part of the circuit surpasses all others from the circumstance that the product of its conductivity and its section is far smaller than in the others, it will be the most adapted to reveal, by the magnitude of its dip, the differences of the electric force at its various points. If its actual length is, at the same time, not inferior to those of the other parts, its reduced length will far surpass those of the other parts; and it is easily conceived that such a relation between the various parts can be brought about, that even its reduced length may remain far greater than the sum of the reduced lengths of all the other parts. But in this case the reduced length of this one part is nearly equal to the reduced lengths of the entire circuit, so that we may substitute, without committing any great error, $\frac{(l)}{(\chi)(\omega)}$ for L, if (l) represent the actual length of the said part, (χ) its conductivity, and (ω) its section; but then the dip of this part changes nearly into $\frac{A}{(l)}$, whence it follows *that the difference of the electrical forces at the extremities of this part is nearly equal to the sum of all the tensions existing in the circuit.* All the tensions seem, as it were, to tend towards this one part causing the electrical separation to appear in it with otherwise unusual energy, when all the tensions, or, at least, the greater part in number and magnitude, are of the same kind.

In this way the scarcely preceptible gradation in the separation of the electricity, in a closed circuit, may be rendered distinctly evident, which otherwise would not be the case without a condenser, on account of the low intensity of voltaic forces.

This remarkable property of voltaic circuits, representing, as it were, their entire nature, had already long since been noticed, in various imperfectly conducting bodies and its origin sought for in their peculiar constitution.[1] The conditions under which this property of the voltaic circuit even in the best conductors, the metals, have however been stated in a letter by the author to the editor of the *Annalen der Physik*;[2] and the necessary

[1] Gilbert's Annalen, vol. viii. pp. 205, 207, and 456; vol. x. p. 11.

[2] Jahrgang, 1826, Part V. p. 117.

precautions, founded on experience, by which the success of the experiment is assured, described in it, are in perfect accordance with the present considerations.

The expression $\frac{A}{L} \cdot \frac{(\lambda)}{(l)}$ denoting the dip of any portion of the circuit vanishes when L is indefinitely great, while A and $\frac{\lambda}{l}$ retain finite values. Consequently, if L assumes an indefinitely great value, while A remains finite, the dip of the straight lines representing the separation of the electricity, in all such parts of the circuit, whose reduced length has a finite ratio to the actual length, vanishes, or what comes to the same thing, the electricity is of equal force at all places of each such part. Now, since L represents the sum of the reduced lengths of all the parts of the circuit, and these reduced lengths evidently can only assume positive values, L becomes indefinite as soon as one of the reduced lengths assumes an infinite value.

Further, since the reduced length of any part represents the quotient obtained by dividing the actual length by the product of the conductivity and the section of the same part, it becomes infinite when the conductivity of this part vanishes; i. e., when this part is a non-conductor of electricity. *When, therefore, a part of the circuit is a non-conductor, the electricity expands uniformly over each of the other parts, and its change from one part to the other is equal to the whole tension there situated.*

This separation of the electricity, relative to the open circuit, is far simpler than that in the closed circuit, which has hitherto formed the object of our consideration as is geometrically represented by the lines F G, H I, K L (Fig. 3), taking a position parallel to A D.

It distinctly demonstrates *that the difference between the electrical forces, occurring at any two places of the circuit, is equal to the sum of all the tensions situated between these two places, and consequently increases or decreases exactly in the same* *proportion as this sum. When, therefore, one of these places is touched abductively, the sum of all the tensions situated between the two marks its appearance at the other place ;* at the same time the direction of the tensions must always be determined by an advance from the latter place.

All the experiments on the open circuit battery, with the help of the electroscope, instituted at such length by Ritter, Erman, and Jäger, and described in Gilbert's Annalen, [1] are expressed in this last law.

All the electroscopic actions of a voltaic circuit of the kind described at the outset have been above stated; we therefore pass for the present to the consideration of the current originating in the circuit, the nature of which, as explained above, is expressed at every point of the circuit by the equation

$$S = \frac{A}{L}.$$

Both the form of this equation as well as the mode by which we reach it, show directly *that the magnitude of the current*

[1] Vol. viii., xii., and xiii.

in such a voltaic circuit remains the same at all points of the circuit, and is solely dependent on the mode of separation of the electricity, so that it does not vary, even though the electric force at any place of the circuit be changed by abductive contact, or in any other way.

This equality of the current at all points of the circuit has been proved by the experiments of Becquerel,[1] and its independency of the electric force at any determinate place of the circuit by those of G. Bischof.[2]

Neither abduction nor addition alters the current of the voltaic circuit as long as they only act directly on a single point of the circuit; but if two different places were acted upon simultaneously, a second current would be formed, which would necessarily, according to circumstances, more or less change the first.

[3] [One grain of water, acidulated to facilitate conduction, will require an

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[1] Bulletin Universal. Physique. Mai, 1825.

[2] Kastner's Archiv, vol. iv. Part I.

[3] Exper. Researches: Faraday. Note 853, Series VII.

electric current to be continued for three minutes and three-quarters of time, to effect its decomposition, which current must be powerful enough to retain a platina wire $\frac{1}{104}$ of an inch in thickness, red hot, in the air during the whole time.

I have not stated the length of wire used, because I find by experiment, as would be expected in theory, that it is indifferent. The same quantity of electricity which passed in a given time can heat an inch of platina wire of a certain diameter red hot, can also heat a hundred, a thousand, or any length of the same wire to the same degree, provided the cooling circumstances are the same for every part in all cases.]

Current Changes with Variations of Electromotive Force or Resistance.—The equation

$$S = \frac{A}{L}$$

shows that the current of a voltaic circuit is subjected to a change, by each variation originating either in the magnitude of a tension or in the reduced length of a part, which latter is itself again determined, both by the actual length of the part as well as by its conductivity and by its section.

[With a given conductor joining two points, it is found by experiment that upon doubling the difference of potential between the points, twice as strong a current flows as before; in other words, with a constant resistance, the current is simply proportional to the E. M. F. or difference of potential between the points. Again it is found that keeping the difference of potential constant, and keeping the section and material of the conducting wire constant, but doubling its length, we halve the current that flows, and generally that if the E. M. F. and section and material of the wire be kept constant, the current will be inversely proportional to the length of the conductor. Again, keeping the E. M. F. length, and material, all constant, the current is halved, by halving the area of the cross-section of the wire.

Consequently, if we define *resistance* as proportional to the length of wire of constant section, and as inversely proportional to the cross-section where that varies, we shall be justified in saying that with a given difference of potentials or E. M. F. between two points, the current which flows will be inversely proportional to the resistance separating these points; and again, that with a constant resistance separating two points, the current flowing will be simply proportional to the E. M. F. or difference of potential between the points. If, then, we call C the current, I the electromotive force, and R the resistance of the conductor, we find that C is proportional to the quotient $\frac{I}{R}$ and is affected by no other circumstances, hence we have

$$C = \frac{I}{R}, \text{ or } R = \frac{I}{C}, \text{ or } I = C\,R.$$

This equation expresses Ohm's law, which may be stated thus: —

When a current is produced in a conductor by an E. M. F., the ratio of the E.

M. F. to the current is independent of the strength of the current, and is called the resistance of the conductor.

This definition of resistance would not be justified if we did not always obtain one and the same value for R in any one conductor, whatever electromotive force may be employed to force a current through it. Electricity and Magnetism: Jenkin, 1873, pp. 81, 82.]

This variety of change may be limited, by supposing only one of the enumerated elements to be variable, and all the remainder constant. We thus arrive at distinct forms of the general equation corresponding to each particular instance of the general capability of change of a circuit.

To render the meaning of this phrase evident by an example, I will suppose that in the circuit only the actual length of a single part is subjected to a continual change, but that all the other values denoting the magnitude of the current remain constantly the same, and consequently also in its equation. If we designate by x this variable length, and the conductivity corresponding to the same part by χ, its section by ω, and the sum of the reduced lengths of all the others by $\varLambda$, so that $L = \varLambda + \frac{x}{\chi . \omega}$, then the general expression for the current changes into the following:—

$$S = \frac{A}{\varLambda + \frac{x}{\chi . \omega}};$$

or if we multiply both the numerator and denominator by $\chi\,\omega$, and substitute a for $\chi\,\omega$ A, and b for $\chi\,\omega\,\varLambda$, into the following:—

$$S = \frac{a}{b + x},$$

where a and b represent two constant magnitudes, and x the variable length of a portion of the circuit fully determined with respect to its substance and its section.

This form of the general equation, in which all the invariable elements have been reduced to the smallest number of constants, is that which the author has practically deduced from experiments to which the theory here developed owes its origin.[1]

The law which it expresses relative to the length of conductors differs essentially from that which Davy formerly and Becquerel more recently were led to by experiments; it also differs very considerable from that advanced by Barlow, as well as from that which I myself had previously drawn from other experiments, although the two latter had come much nearer the truth. The first, in fact, is nothing more than a formula of interpolation, which is valid only for a relatively very short variable part of the entire circuit, and, nevertheless, is still applicable in quite different possible modes of conduction, which is already evident, from its merely admitting the variable portion of the circuit, and leaving out of consideration all the other part; but all partake in common of this evil, that they have admitted a foreign source of variability, produced by the chemical change of the fluid

[1] See Schweigger's Jahrbuch, 1826. Part II.

portion of the circuit, which will be more fully treated of hereafter.

I have already, in other places, treated more at length of the relations of the various forms of the law to one another.

From the numerous separate peculiarities of the voltaic circuit resulting from the general equation $S = \frac{A}{L}$, I will here mention merely a few.

Distinction between Thermo and Hydro Electric Circuits. — It is immediately evident that a change in the arrangement of the parts has no influence on the magnitude of the current if the sum of the tensions be not affected by it.

Nor is the magnitude of the current altered when the sum of the tensions and the entire reduced length of the circuit change in the same proportion; consequently a circuit, the sum of whose tensions is very small in comparison to that of another circuit, may still produce a current which in energy may be equal to that in the other circuit, when that which it loses in force of tension (electromotive force) is replaced by a shortening of its reduced length (resistance).

In this circumstance is the source of the peculiar difference between thermo and hydro electric circuits. In the former only metals occur as parts of the circuit; in the latter, besides the metals, aqueous fluids, whose power of conduction, in comparison to that of the metals, is exceedingly small; on which account the reduced lengths of the fluid surpass beyond all proportion those of the metallic parts, although in all respects their dimensions are equal, and even remain considerably greater when diminished by shortening their actual lengths, and increasing their sections, so long, at least, as this diminution is not carried too far. And thence it is that the reduced length of the thermo-circuit is, in general, far smaller than that of the hydro-circuit, whence we may infer a tension smaller in the same proportion in the former, although the magnitude of the current in the thermo-circuit is not inferior to that in the hydro-circuit.

The great difference between a thermo

and a hydro circuit, both of which produce a current of the same energy, is evident when the same change is made on both, as will be shown in the following consideration:—

Let the reduced length of a thermo-circuit be L, and the sum of its tensions A, the reduced length of an hydro-circuit m L, and the sum of its tensions m A, then the magnitude of the current in the former is expressed by $\frac{A}{L}$, in the latter by $\frac{m\,A}{m\,L}$, and is consequently the same in both circuits. But this equality no longer exists if the same new part λ of the reduced length be introduced into both, for then the magnitude of the current is in the first $\frac{A}{L + \lambda}$, and in the second $\frac{m\,A}{m\,L + \lambda}$.

If we connect with this determination a valuation, even if merely superficial, of the quantities m, L and λ, we shall readily be convinced that in cases where the simple hydro-circuit can still produce in the part λ actions of heat or chemical decomposition, the simple thermo-circuit may not possess the hundredth, and in some cases not the thousandth, part of the requisite force, whence the absence of similar effects in it is easily to be understood.

We are also able to understand why a diminution of the reduced lengths of the thermo-circuit (by increasing, for instance, the section of the metals constituting it) cannot give rise to the production of those effects, although the strength of its current may be increased by this means to a higher degree than in the hydro-circuit producing such effects. This difference in the conductivity of metallic bodies and aqueous fluids is the cause of a peculiarity appearing in the case of hydro-circuits, which perhaps may be properly mentioned here.

Under ordinary circumstances, the reduced length of the fluid portion is so large, in comparison with that of the metallic portion, that the latter may be ignored, and the former alone taken instead of the reduced length of the entire circuit; but then the magnitude of the current in circuits which have the same tension is in the inverse ratio to the reduced length of the fluid portion.

Consequently if such circuits only are compared, in which the fluid parts have the same actual lengths and the same conductivities, *then the magnitude of the current in these circuits is in direct ratio to the section of the fluid portion.*

However, it must not be overlooked that a more complex definition must supersede this simple one when the reduced length of the metallic portion can no longer be regarded as evanescent towards that of the fluid, which case occurs whenever the metallic portion is very long and thin, or the fluid portion is a good conductor, and with unusually large terminal surfaces.

From the equation $S = \frac{A}{L}$ we can easily see that, when a portion is taken from the voltaic circuit, and is replaced by another, and after this change the electromotive force as well as the strength of the current remains still the same, these two parts have the same resistance; *consequently their actual lengths are as the products of their conductivities and sections. The actual lengths of such parts are therefore, when they have like sections, as their conductivities, and when they have like conductivities as their sections.*

By the first of these two relations we are enabled to determine the conductivities of various bodies in a manner which is a great improvement over the previously mentioned process, and it has already been employed by Becquerel and myself for several metals.[1]

The second relation may serve to demonstrate experimentally the independence of the effect on the form of the section as has previously been done by Davy, and recently by myself.[2]

The Electromotive Force and Resistance of a Battery depends on the Number of Ele-

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[1] Bulletin Universel Physique, Mai, 1825, and Schweigger's Jahrbuch, 1826. Part II.

[2] Gilbert's Annalen, nn. Folge, vol. xi., p. 253, and Schweigger's Jahrbuch, 1827.

ments. (*In series.*) — In the voltaic pile the sum of the tensions and the reduced length of the simple circuit is repeated as frequently as the number of elements of which it consists.

[Under no circumstances do we obtain in the form of a current the whole of the electricity produced by the chemical actions going on in the battery. The amount of electricity realized, or, in other words, the force of the current, is equal to the sum of the electromotive forces divided by the sum of the resistances of the circuit. Thus let F denote the actual force of the current, that is, its power to produce heat magnetism, chemical action, or any of its other effects; E the electromotive force, and R the resistance of the wires and liquids, then $F = \frac{E}{R}$.

The different causes which influence the quantity of electricity obtained in a voltaic circuit were investigated mathematically by Professor Ohm of Nuremberg, and his formulæ, which have been verified experimentally by Daniell, Wheatstone, and others, may be regarded as the basis on which all the investigations that have since been made relative to the force of the current have been founded.

By increasing the number of elements of a voltaic battery, we increase the tension urging the electricity forward, but at the same time we increase the amount of resistance offered by the liquid portion of the circuit; so that provided in both cases the circuit be completed by a competent conductor, such as a stout copper wire, we obtain in both cases the same results, the electromotive forces and the resistances being increased by an equal amount, for $\frac{E}{R} = \frac{n\,E}{n\,R}$. — Electricity. Noad, revised by Preece, 1879. pp. 198, 199.]

If therefore we designate by A the sum of all the tensions in the simple circuit, by L its reduced length, and by n the number of elements in the battery, the magnitude of the current in the closed battery circuit is evidently $\frac{n\,A}{n\,L}$, while in the simple closed circuit it is $\frac{A}{L}$.

If we now introduce into the simple circuit, as well as into the battery circuit, one and the same new part Λ of the reduced length, upon which the current is to act, the magnitude of the current thus altered in the simple circuit will be $\frac{A}{L + \Lambda}$, and in the voltaic battery circuit

$$\frac{n\,A}{n\,L + \Lambda},\ \text{or}\ \frac{A}{L + \frac{\Lambda}{n}}.$$

It is hence evident *that the current is constantly greater in a voltaic battery than in the simple circuit, but it is merely imperceptibly greater so long as* Λ *is very small in comparison with L; on the contrary, this increase approximates the nearer to* N *times, the greater* Λ *becomes to* N *L, and consequently the more so in comparison with L.*

Besides this mode of increasing the

magnitude of the voltaic current, there is a second one, which consists in shortening the reduced lengths of the simple circuit, which may be effected by increasing its section, or placing several simple circuits by the side of each other, and connecting them in such a way that together they only form one single simple circuit.

[That is, that they shall be in parallel circuit, or in multiple arc with each other. — Ed.]

If now we retain the same signs, so that $\frac{A}{L + \Lambda}$ again denotes the strength of the current in one element, then, in the above-mentioned combination of n elements into a single circuit, the magnitude of the current is evidently

$$\frac{A}{\frac{L}{n} + \Lambda}, \text{ or } \frac{n\,A}{L + n\,\Lambda},$$

which indicates a slight increase in the action of the new combination when Λ is very great in comparison with L; on the *contrary, a very powerful one when Λ is very small in comparison with $\frac{L}{n}$, and consequently still more in comparison with L.*

[This is restated by De La Rive as follows:—

If we increase or diminish the resistance of any part of a circuit, the total intensity of the current diminishes or increases, all other circumstances remaining the same, in a proportion which is the same as that existing between the resistance added or removed, and the total new resistance of the entire circuit.] [1]

It hence follows that the one combination is most active in those cases where the other ceases to be so, and *vice versa.* If, therefore, we have a certain number of simple circuits intended to act upon the portion whose reduced length is Λ, much depends on the way in which they are

[1] Treatise on Electricity: De La Rive, vol. ii., p. 78. 1856.

placed, in order to produce the greatest amount of current: whether all be side by side, or all in succession, or whether part be placed by the side of each other, and part in series.

It may be mathematically shown that it is most advantageous to form them into a voltaic combination, of so many equal parts, that the square of this number be equal to the quotient $\frac{A}{L}$. When $\frac{A}{L}$ is equal to, or smaller than, A, they are best arranged by the side of each other; but should be arranged in series when $\frac{A}{L}$ is equal to, or larger than, the square of the number of all of the elements.

We see in this determination the reason why in most cases a simple circuit, or at least a voltaic combination of only a few simple circuits, is sufficient to produce the greatest effect. If we bear in mind, that since the quantity of the current is the same at all points of the circuit, its intensity at the various points must be in inverse proportion to the magnitude of the section there situated, and if we grant that the magnetic and chemical effects as well as the phenomena of light and heat in the circuit, are direct indications of the electrical current, and that their energy is determined by that of the current itself, then a close analysis of the current will lead us to the perfect explanation of the numerous and puzzling anomalies to be observed in the voltaic circuit, in so far as we are justified in regarding the physical nature of the circuit as invariable.[1]

Action of Galvanometers. — Those great differences which we frequently meet in the statements of various observers, and which do not arise from the dimensions of their different apparatus, have undoubtedly their origin in the double capability of change of the hydro-circuits, and will therefore cease when this circumstance is allowed for on a repetition of the experiments.

The remarkable variability in the action of the same galvanometer in various cir-

[1] See Schweigger's Jahrbuch, 1826, for a more detailed explanation of the separate points. — G. S. O.

cuits, and of different galvanometers in the same circuit, is fully explained by the preceding consideration. For if we denote by A the sum of the tensions, and by L the reduced length of any voltaic circuit, $\frac{A}{L}$ expresses the strength of its current.

If we now imagine a galvanometer of n, similar convolutions each of the reduced length λ,
$$\frac{A}{L + n\lambda}$$
indicates the strength of the current when the galvanometer is made an integral part of the circuit.

Moreover, if we grant, for the sake of simplicity, that each of the n convolutions exerts the same action on the magnetic needle, the action of the multiplying coils on the needle is evidently $\frac{n A}{L + n\lambda}$, when the action of an exactly similar coil of the circuit considered apart is taken as $\frac{A}{L}$.

Hence it follows directly that the action on the magnetic needle is augmented or *weakened by the multiplying coils, according as n L is greater or smaller than $L + \lambda$; i.e.; according as n times the reduced length of the circuit outside of the coils is greater or less than the reduced length of the circuit including such coils.*

Further, a mere glance at the expression by which the action of the galvanometer coils on the needle has been determined, will show that the greatest or smallest action occurs as soon as L may be neglected with reference to $n \lambda$, and is expressed by $\frac{A}{\lambda}$.

If we compare this extreme action of the multiplying coils with that which a perfectly similarly constructed convolution of the circuit external to the galvanometer produces, we perceive that both are in the same ratio to one another as the reduced lengths L and λ, which relation may serve to determine one of the values when the others are known.

The expression found for the extreme action of the multiplying coils shows that it is proportional to the tension (potential) *of the circuit, and independent of its reduced length ;* consequently the extreme action of the same galvanometer may serve not merely to determine the tensions in various circuits, but it also indicates that the extreme action may also be augmented to the same degree as the sum of the tensions is increased, which may be effected by forming a voltaic combination with several simple circuits.

If we represent the actual length of one coil of the galvanometer by l, its conductivity by χ, and its section by ω, so that $\lambda = \frac{l}{\chi . \omega}$, the expression for the extreme action of the galvanometer is converted into

$$\chi . \omega . \frac{A}{l},$$

from which it will be seen *that the extreme action of two galvanometers of different metals, constructed of wire of the same thickness, are in the same ratio to each other as the conductivities of these metals, and that the extreme actions of two galvanometers formed of wire of the same metal, are in the same proportion to each other as the sections of the wires.*

All these various peculiarities of the galvanometer have been shown to be founded on experience — partly on experiments performed by other persons, and partly on those of the author.[1]

The most recent experiments made on this subject on thermo-circuits have, in a different (and, in a certain sense, opposite) manner, already pointed to the conclusion, deduced above from an equation of the reduced lengths, that the sum of the tensions in a thermo-circuit is far weaker than in the ordinary hydro-circuits; and a provisional comparison has convinced me, that with respect to the heating effects, if they are to be predicted with certainty, a voltaic combination of some hundred well-chosen simple thermo-circuits is requisite, and for chemical effects of some energy a far greater apparatus.

Experiments, which place this prediction beyond doubt, will afford a new and not unimportant confirmation to the theory here propounded.

Divided or Derived Circuits. — The pre-

[1] Schweigger's Jahrbuch, 1826, Part II.; and 1827.

vious considerations are also sufficient to indicate the process which is carried on when the voltaic circuit is divided at any place into two or more branches. For this purpose we may recall, that at the time we found the equation $S = \frac{A}{L}$, we also obtained the rule that the strength of the current in any homogeneous part of the voltaic circuit is given by the quotient of the difference between the electrical forces (the potentials) existing at the extremities of the portion, and its reduced length. It is true this rule was only advanced above for the case in which the circuit is not anywhere divided into several branches; but a very simple analogous consideration derived from the equality of the added and deducted quantity of electricity in all sections of each prismatic part, is sufficient to prove that the same rule holds good for every single branch, in case of a division of the circuit. Let us suppose a circuit divided, for instance, into three branches, whose reduced lengths are λ, λ', λ''; and that at each of these places, the undivided circuit and the single branches possess equal electrical force, and consequently no tension occurs there, and designate by a the difference between the electrical forces at these two places; then, according to the above rule, the strength of the current in each of the three branches is

$$\frac{a}{\lambda}, \frac{a}{\lambda'}, \frac{a}{\lambda''};$$

whence it directly follows *that the currents in the three branches are inversely as their reduced lengths;* so that each separately may be found when the sum of all three together is known.

But the sum of all three is evidently equal to the strength of the current at any other place of the non-divided portion of the circuit, for otherwise the permanent state of the circuit, which is still constantly supposed, would not be maintained.

If we connect with this the conclusion resulting from the above considerations; namely, that the magnitude of the current, and the nature of each homogeneous part of the circuit, give the dip of the corresponding straight line representing the separation of the electricity (the fall of potential), we are certain that the figure of the separation belonging to the non-divided portion of the circuit must remain the same as long as the current in it retains the same strength, and *vice versa;* whence it follows that the variability of the current in the non-divided portion necessarily supposes that the difference between the electrical forces at the extremities of this portion is constant.

[An important law is that which regulates the distribution of the electric current between two parallel conductors placed in the circuit.

If they are of the same nature, of the same diameter, and of the same length, — a condition which is realized for instance by two similar wires, — it is evident that the current divides itself equally between them. But if they are of different lengths, still being of the same nature and of the same diameter, let the length of one be m, and that of the other n, then the proportion of the current that traverses each of them is inversely as its length, and the total intensity of the current is the same as if, instead of the two wires of the lengths m and n, a single wire were placed in the circuit of a length $\frac{m\,n}{m+n}$. Generally, if a and λ represent the respective resistances of any two conductors interposed parallel in the circuit, the complete resistance of the two conductors is the same as that of a single conductor the resistance of which might be expressed by $\frac{a\,\lambda}{a+\lambda}$. The two conductors may differ in their nature, their length, and their section, or in these three circumstances together. Only it is necessary that they be both metallic or both liquid.—DE LA RIVE.[1]]

If we now imagine, instead of the separate branches, a single conductor of the reduced length A brought into the circuit

[1] Treatise on Electricity: De La Rive, vol. ii. p. 84. 1856.

which does not at all alter the strength of its current and its tensions, then, according to what has just been stated, the difference between the electrical forces at its extremities must still always remain a and consequently be

$$\frac{a}{\varLambda} = \frac{a}{\lambda} + \frac{a}{\lambda'} + \frac{a}{\lambda''},$$

or

$$\frac{l}{\varLambda} = \frac{l}{\lambda} + \frac{l}{\lambda'} + \frac{l}{\lambda''},$$

which equation serves to determine the value of $\varLambda$. But if this value is known, and we call A the sum of all the tensions in the circuit, and L the reduced length of the non-divided portion of the circuit, *we obtain, as is known, for the magnitude of the current in the last-mentioned circuit*

$$\frac{\mathrm{A}}{\mathrm{L} + \varLambda},$$

which is equal to the sum of the currents occurring in the separate branches.

Now, since it has already been proved that the currents in the separate branches are in inverse proportion to one another as the reduced lengths of these branches, we obtain for the strength of the current in the branch whose reduced length is λ,

$$\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda};$$

in the branch whose reduced length is λ',

$$\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda'}.$$

and in the branch whose reduced length is λ'',

$$\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda''};$$

This remote and hitherto but slightly noticed peculiarity of the galvanic circuit, I have also found to be perfectly confirmed by experiment.[1]

Herewith is concluded the consideration of such voltaic circuits as have already attained the permanent state, and which neither suffer modification by the influence of the surrounding atmosphere, nor by a gradual change in their chemical composition.

But from this point the simplicity of the subject decreases more and more, so that the previous elementary treatment soon

[1] Schweigger's Jahrbuch, 1827.

entirely disappears. With respect to those circuits on which the atmosphere exercises some influence, and whose condition varies with time, without this change originating in a progressive chemical transformation of the circuit, and is thus distinguished from all the others by the strength of its current being different at different points, I have been content, with respect to each of these, always to treat of only the most simple case, as they but rarely occur in nature, and in general appear to be of less interest.

This plan I have adopted the more willingly, as I intend to return to this subject at some future time.

But with regard to that modification in galvanic circuits which is produced by a chemical change in the circuit, proceeding first from the current, and again reacting on it, I have devoted special attention to it in the Appendix. The course adopted is founded on a vast number of experiments performed on this subject, the communication of which, however, I omit, because they appear to be capable of being far more accurately determined than I was able to do at that time, from failing to attend to several elements in operation; nevertheless, it seems proper to mention the circumstance, in this place, in order that the careful manner with which I advance in the inquiry, and which I regard as being due to truth, may not unduly operate against its reception.

I have sought for the source of the chemical changes caused by the current, in the peculiar mode of separation of the electricity of the circuit described above; and it can scarcely be doubted have found at least the main course.

It is immediately evident that each disk belonging to a section of a voltaic circuit, which obeys the electric attractions and repulsions, and does not oppose their movement, must in the closed circuit be propelled always toward one side only, as these attractions and repulsions, in consequence of the continually varying electric force, are different at the two sides, and it is mathematically demonstrable *that the force with which it is driven to the* *one side is in the ratio compounded of the magnitude of the electric current, and of the electric force in the disk.*

It is true, however, that merely a change of position in space would be immediately produced by this. But if this disk be regarded as a compound body, the constituent parts of which, according to electro-chemical views, are distinguished by a difference in their electrical relation to one another, it thence directly follows that this one-sided pressure on the various constituent parts would in most cases act with unequal force, and sometimes even in contrary direction, and must thus excite a tendency in them to separate from one another.

On the Decomposing Power of a Current. —From this consideration results a distinct activity of the circuit, tending to produce a chemical change in its parts, which I have termed its *decomposing force,* and I have endeavored to determine its value for each particular case.

This determination is independent of the mode in which the electricity may be conceived to be associated with the atoms.[1]

Granting, which seems to be the most natural, that the electricity is diffused proportionately to the mass over the space which these bodies then occupy, a complete analysis will show *that the decomposing forces of the circuit is in direct proportion to the energy of the current, and moreover, that it depends on a coefficient, to be derived from the nature of the constituent parts and their chemical equivalents.*

From the nature of this decomposing force of the circuit, which is of equal energy at all places of an homogeneous portion, it directly follows, that when it is capable of overcoming, under all circumstances, the reciprocal connection of the constituent parts, the separation and removal of the constituents to both sides of the circuit are limited solely by me-

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[1] I shall shortly have occasion to speak of the peculiar import of this remark, when I shall attempt to reduce the actions of the parts of a galvanic circuit on one another, as discovered by Ampère, to the usual electrical attractions and repulsions. — G. S. O.

chanical obstacles; but if the connection of the constituent parts *inter se*, either immediately at the beginning everywhere, or in the course of the action anywhere, overcome the decomposing force of the circuit, then, from that time, no further movement of the elements can take place.

This general description of the decomposing force is in accordance with the experiments of Davy and others.

[When electro-chemical decomposition takes place, there is great reason to believe that the quantity of matter decomposed is not proportionate to the intensity, but to the quantity of electricity passed. — *Ex. Researches*, January, 1833. Note 329.

For this case of electro-chemical decomposition, and it is probable for all cases, it follows that the chemical power, like the magnetic force, is in direct proportion to the absolute quantity of electricity which passes. — *Ex. Researches*. Note 377.

Electro-chemical decomposition is well known to depend essentially upon the *current* of electricity. I have shown that in certain cases the decomposition is proportionate to the quantity of electricity passing, whatever may be its intensity or source; and that the same is probably true for all cases, even when the utmost generality is taken on the one hand, and great precision of expression on the other. — *Ex. Researches*, June, 1833. Note 510. Faraday.]

There is a peculiar state which seems to be produced in most cases of the separation of the two elements of a chemically compound liquid, which is especially worthy of attention, and which is caused in the following manner: —

When the decomposition is confined solely to a limited portion of the circuit, and the constituent parts of the one kind are propelled towards the one side of this part, and those of the other to its opposite side, then for this very reason, a natural limit is prescribed to the action; for the constituent part preponderating on the one side of any disk, within the portion in the act of decomposition, will by force of its innate repulsive power, constantly oppose the movement of a similar constituent to the same side, so that the decomposing force of the circuit has not merely to overcome the constant connection of the two constituents *inter se*, but also this reaction of each constituent on itself

It is hence evident that a cessation in the chemical change must occur, if at any time there arises an equilibrium between the two forces. This state, founded upon a peculiar chemical and permanent separation of the constituents of the portion of the circuit in the act of decomposition, is the very one from which I started, and whose nature I have endeavored to determine as accurately as possible in the Appendix. Even the mere description of the mode of origin of this remarkable phenomenon shows that at the extremities of the divided portion no natural equilibrium can occur, on which account the two constituents must be retained at these two places, by a mechanical force, unless they pass over to the next parts of the circuit, or, where the other circumstances allow, separate entirely from the circuit. Who would not recognize in this statement all the chief circumstances hitherto observed of the external phenomenon in chemical decompositions by the circuit?

If the current, and at the same time the decomposing force, be suddenly interrupted, the separated constituents gradually return to their natural equilibrium; but tend to reassume immediately the relinguished state if the current is re-established. During this process, both the conductivity and the mode of excitation between the elements of the portion in the act of decomposition obviously vary with their chemical nature; but this necessarily produces a constant change in the electrical separation, and in the strengths of the current in the voltaic circuit dependent thereon, which only finds its natural limits in the permanent state of the electrical separation. For the accurate determination of this last stage of the electric current it is requisite to be acquainted with the law which governs the conductivity and force of excitation of the variable mixtures formed of two different liquids.

Experiment has hitherto afforded insufficient data for this purpose; and I have therefore given the preference to a theoretical supposition, which will supply its place until the true law is discovered.

With the help of this law, which is not altogether imaginary, I now arrive at the equations which make known for each case all the individual circumstances constituting the permanent state of the chemical separation in the voltaic circuit; I have, however, neglected the further use of them, as the present state of our experimental knowledge in this respect did not appear to me to repay the requisite trouble.

Nevertheless, in order to compare in their general features the results of this examination with what has hitherto been supplied by experiments, I have fully carried out one particular case, and have found that the formula represents very satisfactorily the kind of wave of the force as I have above described it.[1]

Having thus given a slight outline of the contents of this Memoir, I will now proceed to the fundamental investigation of the individual points.

[1] Schweigger's Jahrbuch, 1826. Part II.

THE VOLTAIC CIRCUIT.

A. GENERAL OBSERVATIONS ON THE DIFFUSION OF ELECTRICITY.

1. A property of bodies, called into activity under certain circumstances, and which we call *electricity*, manifests itself in space, by the bodies which possess it, and which on that account are termed *electric*, either attracting or repelling one another.

In order to investigate the changes which occur in the electric condition of a body A, in a perfectly definite manner, this body is each time brought, under similar circumstances, into contact with a second movable body of invariable electrical condition, called the *Electroscope*, and the force with which the electroscope is repelled or attracted by the body is determined.

This force is termed the *electroscopic* force (potential) of the body A; and to distinguish whether it is attractive or repulsive, we place before the expression for its measure the sign + in the one case, and — in the other.

The same body A may also serve to determine the electroscopic force in various parts of the same body. For this purpose we take the body A of very small dimensions, so that when we bring it into contact with the part to be tested of any third body, it may from its smallness be regarded as a substitute for this part; then its electroscopic force, measured in the way described, will, when it happens to be different at the various places, make known the relative difference with regard to electricity between these places.

The intention of the preceding explanations is to give a simple and determinate signification to the expression "electroscopic force;" it does not come within the limits of our plan to take notice either of the greater or less practicability of this process, nor to compare *inter se* the various possible modes of proceeding for the determination of the electroscopic force.

2. We perceive that the electroscopic force moves from one place to another, so that it does not merely vary at different places at the same time, but also at a single place at different times.

In order to determine in what manner the electroscopic force is dependent upon the time when it is perceived, and on the place where it is elicited, we must set out from the fundamental laws to which the exchange of electroscopic force occurring between the elements of a body are subject.

These fundamental laws are of two kinds, either borrowed from experiment, or, where this is wanting, assumed hypothetically.

The admissibility of the former is beyond all doubt, and the justice of the latter is distinctly evident from the coincidence of the results deduced from calculation, with those which actually occur; for since the phenomenon with all its modifications is expressed in the most determinate manner by calculation, it follows, since no new uncertainties arise and increase the earlier ones during the process, that an equally perfect observation of nature must in a decisive manner either confirm or refute its statements. This, in fact, is the chief merit of mathematical analysis, that it calls forth by its never-vacillating expressions, a generality of ideas, which continually excites to renewed experiments, and thus leads to a more profound knowledge of nature.

Every theory of a class of natural phenomena founded upon facts which will not admit of analytic investigation in the form of its exposition is imperfect; and no reliance is to be placed upon a theory developed in ever so strict a form which is not confirmed to a sufficient extent by observation.

So long, therefore, as not even one portion of the effects of a natural force has been observed with the greatest accuracy in all its gradations, the calculation employed in its investigation only treads an uncertain ground, as there is no touchstone for its hypotheses, and, in fact, it would be far better to wait a more fit time; but when it goes to work with the proper authority, it enriches, at least in an indirect manner, the field it occupies, with new natural phenomena, as universal experience shows. It has seemed necessary to premise these general remarks, as they not only throw much light on what follows, but also because they explain why the voltaic phenomena have not been long since mathematically treated with greater success; although, as we shall subsequently find, the requisite course has been already pursued in another, apparently less prepared, branch of physics.

After these reflections we will now proceed to the establishment of the fundamental laws themselves.

3. When two electrical elements, E and E^1, of equal magnitude, of like form, and similarly placed with respect to each other, but unequally powerful, are situated at the proper distance from each other, they exhibit a mutual tendency to attain electric equilibrium, which is apparent in both constantly and uninterruptedly approaching nearer to the mean of their

electric state, until they have actually attained it.

That is to say, both elements reciprocally change their electric state as long as a difference continues to exist between their electroscopic forces; but this change ceases as soon as they have both attained the same electroscopic force. Consequently, this change of the electric difference of the elements is so dependent that the one disappears with the other.

We now suppose that the change effected in an extremely short instant of time in both elements is proportional to the difference of their contemporaneous electroscopic force and the magnitude of the instant of time; and without yet attending to any material distinctions of the electricity, it is always to be understood that the forces designated by + and — are to be treated exactly as opposite magnitudes. That the change is effected accurately, according to the difference of the forces, is a mathematical supposition, the most natural because it is the most simple; all the rest is given by experiment.

The motion of electricity is effected in most bodies so rapidly that we are seldom able to determine its changes at the various places, and on that account we are not in a condition to discover by observation the law according to which they act. The galvanic phenomena, in which such changes occur in a constant form, are therefore of the highest importance for testing this assumption: for if the conclusions drawn from the supposition are thoroughly confirmed by those phenomena, it is admissible, and may then be applied without any further consideration to all analogous researches, at least within the same limits of force.

We have assumed, in accordance with the observations hitherto made, that when by any two exteriorly like constituted elements, whether they be of the same or of different matter, a mutual change in their electrical state is produced, the one loses just so much force as the other gains. Should it hereafter be shown by experiments, that bodies exhibit a relation similar to that which in the theory of heat is termed the capacity of bodies, the law we have established will have to undergo a slight alteration, which we shall point out in the proper place.

4. When the two elements E and E′ are not of equal magnitude, it is still allowed to regard them as sums of equal parts. Granting that an element E consists of m perfectly equal parts, and the other E′ of m' exactly similar parts, then, if we imagine the elements E and E′ exceedingly small in comparison with their mutual distance so that the distances from each part of the one to each part of the other element are equal, the sum of the actions of all the m' parts of the element E′ on a part of E will be m' times that which a part of E′ exerts on a part of E.

It is thus evident that, in order to ascertain the mutual actions of dissimilar elements on each other, they must be taken as proportional not merely to the difference of their electroscopic forces and their duration, but also to the product of their relative magnitudes.

We shall in future term the sum of the

electroscopic actions, referred to the magnitude of the elements,—by which therefore we have to understand the force multiplied by the magnitude of the space over which it is diffused, in the case where the same force prevails at all places in this space, —*the quantity of electricity*, without intending to determine anything thereby with respect to the material nature of electricity. The same observation is applicable to all figurative expressions introduced, without which, perhaps for good reasons, our language could not exist.

In cases where the elements cannot be regarded as evanescent in comparison with their relative distances, a function, to be determined separately for each given case from their dimensions and their mean distance, must be substituted for the product of the magnitudes of the two elements, and which we will designate, where it is employed, by F.

5. Hitherto we have taken no notice of the influence of the mutual distance of the elements between which an equalization of their electric state takes place, be-

cause as yet we have only considered such elements as always retain the same relative distance. But now the question arises, whether this exchange is directly effected only between adjacent elements, or if it extends to others more distant, and how on the one or the other supposition is its magnitude modified by the distance?

Following the example of Laplace, it is customary in cases where molecular actions at the least distance come into play, to employ a particular mode of representation, according to which a direct mutual action between two elements separated by others still occurs at finite distances, which action, however, decreases so rapidly, that even at any perceptible distance, be it ever so minute, it has to be considered as perfectly evanescent.

Laplace was led to this hypothesis, because the supposition that the direct action only extended to the next element, produced equations, the individual members of which were not of the same dimension relatively to the differentials of the variable quantities, — a non-uniformity which is opposed to the spirit of the differential calculus. This apparent unavoidable disproportion between the members of a differential equation, belonging nevertheless necessarily to one another, is too remarkable not to attract the attention of those to whom such inquiries are of any value; an attempt therefore to add something to the explanation of this enigma will be the more proper in this place, as we gain the advantage of rendering thereby the subsequent considerations more simple and concise.

We shall merely take as an instance the propagation of electricity, and it will not be difficult to transfer the obtained results to any other similar subject, as we shall subsequently have occasion to demonstrate in another example.

6. Above all, it is requisite that the term goodness of conduction be accurately defined. But we express the energy of conduction between two places by a magnitude which, under otherwise similar circumstances, is proportional to the quan-

tity carried over in a certain time from one place to the other, multiplied by the distance of the two places from each other.

If the two places are extended, then we have to understand by their distance the straight line connecting the centres of the dimensions of the two places. If we transfer this idea to two electric elements, E and E′, and call s the mutual distance of their centres, ϱ the quantity of electricity, which under accurately determined and invariable circumstances is carried over from one element to the other, and χ the conductivity between them,

$$\chi = \varrho \,.\, s.$$

We will now endeavor to determine more precisely the quantity of electricity, denoted by ϱ. According to Sect. 4, the quantity of electricity which is transferred in an exceedingly short time from one element to the other is, the distance being invariable, in general proportional to the difference between the electroscopic forces, the duration, and the size of each of the two elements. If, therefore, we designate the electroscopic forces of the two elements E and E′ by u and u', and the space they occupy by m and m', we obtain for the quantity of electricity carried over from E′ to E in the element of time $d\ t$ the following expression:—

$$\alpha\ m\ m'\ (u'-u)\ d\ t,$$

where α represents a coefficient depending in some way on the distance s. This quantity changes every moment if $u'-u$ is variable; but if we suppose that the forces u' and u remain constant at all times, it merely depends on the magnitude of the instant of time $d\ t$, we can consequently extend it to the unity of time; if we place the present constant difference of the forces $u'-u$ equal to the unity of force, it then becomes

$$\alpha\ m\ m'.$$

This quantity of electricity is for the two elements E and E′ whose position is invariable, constant under the same circumstances, on which account it may be employed in the determination of the power of conduction just mentioned. For if we understand by ϱ the quantity of electricity transferred from E′ to E in the unity of time, with a constant difference of the electroscopic forces equal to the unity of force, we have

$$\varrho = \alpha\, m\, m', \text{ and then}$$
$$\chi = \alpha\, m\, m'\, s.$$

If we take from this last equation the value of $\alpha\, m\, m'$ and substitute it in the expression

$$\alpha\, m\, m'\,(u' - u)\, d\, t,$$

we obtain for the variable quantity of electricity which passes over in the instant of time $d t$ from E′ to E, the following:—

$$\frac{\chi\,(u' - u)\, d\, t}{s} \qquad (\delta)$$

which expression is not accompanied by the above-mentioned disproportion between the members of the differential equation, as will soon be perceived.

7. The course hitherto pursued was based upon the supposition that the action exerted by one element on the other is proportional to the product of the space occupied by the two elements, an assump-

tion which, as was already observed in Sect. 4, can no longer be allowed in cases where it is a question of the mutual action of elements situated indefinitely near each other, because it either establishes a relation between the magnitudes of the elements and their mutual distances, or prescribes to these elements a certain form. The previously found expression (♂) for the variable quantity of electricity passing from one element to the other, possesses therefore no slight advantage in being entirely independent of this supposition; for whatever may have to be placed in any determinate case instead of the product $m\,m'$, the expression (♂) constantly remains the same, this peculiarity being referable solely to the power of conduction χ. If, for instance, F designate, as was stated in Sect. 4, the function corresponding to such a case, of the dimensions, and of the mean distance of both elements, the expression $\alpha\, m\, m'\, (u' - u)\, d\, t$ not merely changes apparently into $\mathrm{F}\,(u' - u)\, d\, t$, but also the equation $\chi = d\, m\, m'\, s$ into the other, $\chi = \mathrm{F}.\, s$, (⊙)

so that if we take the value of F from this equation and place it in the above expression, we always obtain

$$\frac{\chi\,(u' - u)\,d\,t}{s}.$$

Moreover, the circumstance of the expression (♂) still remaining valid for corpuscles, whose dimensions are no longer indefinitely small, is of some importance when the same electroscopic force only exists merely at all points of each such part. It is hence evident how intimately our considerations are allied to the spirit of the differential calculus; for uniformity in all points with reference to the property which enters into the calculation is precisely the distinctive characteristic required by the differential calculus from that which it is to receive as an element.

If we institute a more profound comparison between the process originating with Laplace, and that here advanced, we shall arrive at some interesting points of comparison.

If, for instance, we consider that for

infinitely small masses at infinitely short distances all particular relations must necessarily have the same weight as for finite masses at finite distances, it is not directly evident how the method of the immortal Laplace — according to which the elements must be constantly treated as if they were placed at finite distances from each other, — could, nevertheless, still afford correct results; but we shall find on closer examination that it acts in fact otherwise than it expresses.

Indeed, since Laplace, when determining the changes of an element by all surrounding it, makes the higher powers of the distance disappear compared with the lower, he therewith assumes, quite in the spirit of the differential calculus, the difference of action itself to be infinitely small, but terms it finite, and treats it also as such; whence it is immediately apparent that he in fact treats that which is infinitely small at an infinitely short distance as finite.

Disregarding, however, the great certainty and distinctness which accompany our manner of representation, there might still be something more to say, and perhaps with some justice, against Laplace's mode of treatment in favor of ours, in this respect, that the former takes not the least account of the possible nature of the *given* elements of bodies, but merely has to do with imaginary elements of space, by which the physical nature of the bodies is almost entirely lost sight of. We may, to render our assertion intelligible by an example, undoubtedly imagine bodies in nature which consist only of homogeneous elements, but whose position to each other, taken in one direction, might be different than when in another direction; such bodies, as our mode of representation immediately shows, might conduct the electricity in one direction in a different manner than in another, notwithstanding that they might appear uniform and equally dense.

In such a case, did it occur, we should have to take refuge, according to Laplace, in considerations which have remained entirely foreign to the general process.

On the other hand, the mode in which bodies conduct affords us the means by which we are enabled to judge of their internal structure, which, from our almost total ignorance of the subject, cannot be immediately shown. Lastly, we may add, that this, our hitherto developed mode of molecular actions, unites in itself the two advanced by Laplace and by Fourier in his theory of heat, and reconciles them with each other.

8. We need now no longer hesitate about allowing the electrical action of an element not to extend beyond the adjacent surrounding elements, so that the action entirely disappears at every finite distance, however small. The extremely limited circle of action with the almost infinite velocity with which electricity passes through many bodies might indeed appear suspicious; but we did not overlook on its admission that our comparison in such cases is only effected by an imaginary relative standard, which is deceitful, and does not therefore warrant us to vary a law so simple and indepen-

dent, until the conclusions drawn from it contradict nature, which in our subject does not seem to be the case.

The sphere of action we have thus fixed, has, although it is infinitely small, precisely the same circumference as that introduced by Laplace, and called finite, where he lets the higher powers of the distance vanish compared with the lower, the reason of which may be found in what has been stated above. The supposition of a finite distance of action in our sense would correspond to the case where Laplace still retains higher powers of distance together with the lower.

9. The bodies on which we observe electric phenomena are in most cases surrounded with the atmosphere; it is therefore requisite, in order to investigate profoundly the entire process, not to disregard the changes which may be produced by the adjacent air. According to the experiments left us by Coulomb on the diffusion of electricity in the surrounding atmosphere, the loss in force thus occasioned is (during a very short constant

time) at least when the intensities are not very considerable, on the one hand proportional to the energy of the electricity, and on the other is dependent on a coëfficient varying according to the contemporaneous nature of the air, but otherwise invariable for the same air.

The knowledge of this enables us to bring the influence of the atmosphere on voltaic phenomena, into calculation wherever it might be requisite. It must, however, not be overlooked here, that Coulomb's experiments were made on electricity which had entered into equilibrium, and was no longer in the process of excitation, with respect to which both observation and calculation have convinced us that it is confined to the surface of bodies, or merely penetrates to a very slight depth into their interior; for from thence may be drawn the conclusion, of some importance with respect to our subject, that all the electricity present in those experiments may have been directly concerned in the transference to the atmosphere.

If we now connect with this observation the law just announced, according to which two elements, situated at any finite distance from each other, no longer exert any direct action on each other, we are justified in concluding that where the electricity is uniformly diffused throughout the entire mass of a finite body, or at least so that proportionately but a small quantity resides in the vicinity of the surface, which case in general occurs when it has entered into motion, the loss which is occasioned by the circumambient air can be but extremely small in comparison to that which takes place when the entire force is situated immediately at the surface, which invariably happens when it has entered into equilibrium; and thence, therefore, it happens that the atmosphere exerts no perceptible influence on voltaic phenomena in the closed circuit when this is composed of good conductors, so that the changes produced by the presence of the atmosphere in phenomena of contact-electricity may be ignored in such cases.

This conclusion, moreover, receives new support from the circumstance, that in the same cases the contact-electricity only remains during an exceedingly short time in the conductors, and even on that account would only give up a very slight portion to the air, even if it were in immediate contact with it.

Although, from what has been stated, it is placed beyond all doubt that the action of the atmosphere has no perceptible influence on the magnitude of effect of the usual voltaic circuits, it is not intended to admit the reverse of the conclusion; viz., that the voltaic conductor exerts no perceptible influence on the electric state of the atmosphere; for mathematical investigation teaches us that the electroscopic action of a body on another has no direct connection with the quantity of electricity which is carried over from one to the other.

10. We arrive at last at that position founded on experiment, and which is of the highest importance for the whole of natural philosophy, since it forms the basis of all the phenomena to which we apply the name of voltaic: it may be expressed thus: Different bodies which touch each

other constantly preserve at the place of contact the same difference between their electroscopic forces by virtue of a contrariety proceeding from their nature, which we are accustomed to designate by the expression *electric tension*, or *difference of bodies*. Thus announced, the position stands, without losing any of its simplicity, in all the generality which belongs to it; for we are nearly always referred to it by every single phenomenon.

Moreover, the above expression is adopted in all its generality, either expressly or tacitly, by all philosophers in the explanation of the electroscopic phenomena of the voltaic battery.

According to our previously developed ideas respecting the mode in which elements act on one another, we must seek for the source of this phenomenon in the elements directly in contact, and consequently we must allow the abrupt transition to take place from one body to the other in an infinitely small extent of space.

11. This being established, we will now proceed to the subject, and in the first

place consider the motion of the electricity, in a homogeneous cylindric or prismatic body, in which all points throughout the whole extent of each section, perpendicular to its axis, possess contemporaneously equal electroscopic force, so that the motion of the electricity can only take place in the direction of its axis. If we imagine this body divided by a number of such sections into disks of infinitely small thickness, and so that in the whole circumference of each disk the electroscopic force does not vary sensibly for each pair of such disks, the expression ♂ given in Sect. 6, can be applied to determine the quantity of electricity passing from one disk to the other; but by the limitation of the distance of action to only the infinitely small distances mentioned in the preceding paragraph, its nature is so modified that it disappears as soon as the divisor ceases to be infinitely small.

If we now choose one of the infinite number of sections invariably for the origin of the abscissæ, and imagine anywhere a second whose distance from the first we denote by x, then $d\,x$ represents the thickness of the disk there situated, which we will designate by M. If we conceive this thickness of the disk to be of like magnitude at all places, and term u the electroscopic force present at the time t in the disk M, whose abscissa is x, so that therefore u in general will be a function of t and x; if we further suppose u' and u_1 to be the values of u when $x + d\,x$, and $x - d\,x$ are substituted respectively for x, then u' and u_1 evidently express the electroscopic forces of the disks situated next the two sides of the disk M, of which we will denote the one belonging to the abscissa $x + d\,x$ by M′, and that belonging to the abscissa $x - d\,x$, by M_1; and it is clearly evident that the distance of the centre of each of the disks M′ and M_1, from the centre of the disk M is $d\,x$. Consequently, by virtue of the expression (δ) given in Sect. 6, if χ represents the conducting power of the disk M′ to M,

$$\frac{\chi\,(u' - u)\,d\,t}{d\,x}$$

is the quantity of electricity which is transferred during the interval of time $d\,t$ from the disk M′ to the disk M, or from the latter to the former, according as $u' - u$ is positive or negative.

In the same manner, when we admit the same power of conduction between M_1 and M,

$$\frac{\chi\,(u_1 - u)\,d\,t}{d\,x}$$

is the quantity of electricity passing over from M_1 to M, when the expression is positive, and from M to M_1 when it is negative.

The total change of the quantity of electricity which the disk M undergoes from the motion of the electricity in the interior of the body in the particle of time $d\,t$, is consequently

$$\frac{\chi\,(u' + u_1 - 2\,u)\,d\,t}{d\,x}$$

and an increase in the quantity of electricity is denoted when this value is positive, and when negative a diminution of the same.

But according to Taylor's theorem

$$u' = u + \frac{d\,u}{d\,x} \cdot d\,x + \frac{d^2\,u}{d\,x^2} \cdot d\,\frac{x^2}{2} + \ldots,$$

and in the same way

$$u' = u' - \frac{d\,u}{d\,x} \cdot d\,x + \frac{d^2\,u}{d\,x^2} \cdot \frac{d\,x^2}{2} - \ldots;$$

consequently

$$u' + u_1 = 2\,u + \frac{d^2\,u}{d\,x^2}\,d\,x^2.$$

According to this the expression just found for the total change of the quantity of electricity present in the disk M is converted during the time $d\,t$ into

$$\chi \cdot \frac{d^2\,u}{d\,x^2}\,d\,x\,d\,t,$$

where χ represents the power of conduction which prevails from one disk to the adjacent one, which we suppose to be invariable throughout the length of the homogeneous body.

It must here be observed that this

valve χ is, on account of the infinitely small distance of action, proportional to the section of the cylindric or prismatic body; if, therefore, we denote the magnitude of this section by ω, and separate this factor from the value χ, always calling the remaining portion χ, the former expression changes into the following:

$$\chi \, \omega \, \frac{d^2 u}{d x^2} \, d x, \, d t,$$

in which χ now represents the conductivity of the body independent of the magnitude of the section, which we will term the *absolute* conductivity of the body in opposition to the former, which may be called the *relative*.

Henceforward wherever the word *conductivity* occurs without qualification, the absolute conductivity is always to be understood.

Hitherto we have not considered the change which the disk suffers from the adjacent atmosphere. This influence may easily be determined. If, for instance, we designate by c the circumference of the disk belonging to the abscissa x, then $c\,d\,x$ is the portion of its surface which is exposed to the air; consequently, according to the experiments of Coulomb, mentioned in section 9, $b\,c\,u\,d\,x\,d\,t$ is the change of the quantity of electricity which is occasioned in the disk M by the passing off of the electricity into the atmosphere during the moment of time $d\,t$, where b represents a coefficient dependent on the contemporaneous nature of the atmosphere, but constant for the same atmosphere.

It expresses a decrease when u is positive, and an increase when u is negative. But in accordance with our original supposition, this action cannot occasion an inequality of the electroscopic force in the same section of the body; or at least, this inequality must be so slight that no perceptible alteration is produced in the other quantities — a circumstance which may nearly always be supposed in the voltaic circuit.

Accordingly the entire change which

the quantity of electricity in the disk M undergoes in the moment of time $d\,t$ is

$$\chi\,\omega\,\frac{d^2 u}{d\,x^2}\,d\,x.\,d\,t - b\,c\,u\,d\,x\,d\,t,$$

in which the portion is comprised which arises from the motion of the electricity in the interior of the body as well as that which is caused by the circumambient atmosphere.

But the entire change of the electroscopic force u in the disk M effected in the moment of time $d\,t$ is

$$\frac{d\,u}{d\,t}\,d\,t,$$

consequently the total change in the quantity of electricity in the disk M during the time $d\,t$ is

$$\omega\,\frac{d\,u}{d\,t}\,d\,x = d\,t,$$

where, however, it is supposed that under all circumstances similar changes in the electroscopic force correspond to similar changes in the quantity of electricity.

If observation showed that different bodies of the same surface underwent a different change in their electroscopic force by the same quantity of electricity, then there would still remain to be added a coefficient γ corresponding to this property of the various bodies. Experience has not yet decided respecting this supposition borrowed from the relation of heat to bodies.

If we assume the two expressions just found for the entire change in the quantity of electricity in the disk M during the moment of time $d\,t$ to be equal, and divide all the members of the equation by $\omega\, d\,x\, d\,t$, we obtain

$$\gamma \frac{d\,u}{d\,t} = \chi \frac{d^2\,u}{d\,x^2} - \frac{b\,c}{\omega}\,u, \qquad (a)$$

from which the electroscopic force u has to be determined as a function of x and t.

12. We have in the preceding paragraph found for the change in the quantity of electricity occurring between the disks M′ and M during the time $d\,t$

$$\frac{\chi\,(u' - u)\,d\,t}{d\,x},$$

and have seen that the direction of the

passage is opposed to the course of the abscissæ when the expression is positive; on the contrary, it proceeds in the direction of the abscissæ when it is negative. In the same way the magnitude of the transition between the disks M_1 and M, when we retain the same relation to its direction, is

$$\frac{\chi\,(u_1 - u)\,d\,t}{d\,x},$$

If we substitute in these expressions for u_1 and u' the transformations given in the same paragraph, and at the same time $\chi\,\omega$ for χ, viz., the absolute power of conduction for the relative, we obtain in both cases

$$\chi\,\omega\,\frac{d\,u}{d\,x}\,d\,t,$$

whence it results that the same quantity of electricity which enters from the one side into the disk M during the element of time $d\,t$ is again in the same time expelled from it towards the other side.

If we imagine this transmission of the electricity occurring at the time t in the disk belonging to the abscissa x, of invariable energy reduced to the unity of time, call it the *electric current*, and designate the magnitude of this current by S, then

$$S = \chi \omega \frac{du}{dx}; \qquad (b)$$

and in this equation positive values for S show that the current takes place opposed to the direction of the abscissæ; negative, that it occurs in the direction of the abscissæ.

13. In the two preceding paragraphs we have constantly had in view a homogeneous prismatic body, and have inquired into the diffusion of the electricity in such a body, on the supposition that throughout the whole extent of each section, perpendicular to its length or axis, the same electroscopic force exists at any time whatsoever. We will now take into consideration the case where two prismatic bodies A and B, of the same kind, but formed of different substances, are adjacent, and touch each other in a common surface.

If we establish for both A and B the same origin of abscissæ, and designate the electroscopic force of A by u, that of B by u', then both u and u' are determined by the equation (a) in paragraph 11, if χ only retain the value each time corresponding to the peculiar substance of each body; but u represents a function of t and x, which holds only so long as the abscissa x corresponds to points in the body A; on the other hand, u' denotes a function of t and x, which holds only when the abscissa x corresponds to the body B.

But there are still some other conditions at this common surface which we will now explain. If we denote for this purpose the separate values of u and u', which they first assume at the common surface, by enclosing the general ones between crotchets, we find according to the law advanced in paragraph 10 the following equation between these separate values:

$$(u) - (u') = a,$$

where a represents a constant magnitude otherwise dependent on the nature of the two bodies.

Besides this condition, which relates to the electroscopic force, there is still a second, which has reference to the electric current. It consists in this, that the electric current in the common surface must in the first place possess equal magnitude and like direction in both bodies, or, if we retain the common factor ω,

$$\chi\,\omega\left(\frac{d\,u}{d\,x}\right)=\chi'\,\omega\left(\frac{d\,u'}{d\,x}\right),$$

where χ represents the actual power of conduction of the body A, χ' that of the body B, and $\left(\frac{d\,u}{d\,x}\right)$, $\left(\frac{d\,u'}{d\,x}\right)$ the particular values of $\frac{d\,u'}{d\,x}$, $\frac{d\,u'}{d\,x}$ immediately belonging to them at the common surface, and in which it was assumed that the origin of the abscissæ was not taken on the common surface. The necessity of this last equation may easily be conceived; for were it otherwise, the two currents would not be of equal energy in the common surface, but there would be more conveyed from the one body to this surface than would be abstracted from it by the other; and if this difference were a finite portion of the entire current, the electroscopic force would increase at that very place, and indeed, considering the surprising fertility of the electric current, would arrive in the shortest time to an exceedingly high degree, as observation has long since demonstrated.

Nor can a smaller quantity of electricity be imparted from the one body to the common surface than it is deprived of by the other, as this circumstance would be evinced by an infinitely high degree of negative electricity.

It is not absolutely requisite for the validity of the preceding determinations that the two bodies in contact have the same base. The section in the one prismatic body may be different in size and form to that in the other, if this does not render the electroscopic force sensibly different at the various points of the same section, which, considering the great energy with which the electricity tends to equilibrium, will not be the case when the bodies are good conductors, whose length far surpasses their other dimensions. In this case everything remains as before, only that the section of the body B must everywhere be distinguished from that of A; consequently the second conditional equation for the place where the two bodies are in contact changes into the following: —

$$\chi\,\omega\left(\frac{d\,u}{d\,x}\right)=\chi'\,\omega'\left(\frac{d\,u'}{d\,x}\right),$$

where ω still represents the section of A, but ω' that of the body B, which at present differs from the former.

There may even exist in the prolongation of the body A two prismatic bodies B and C, separated from each other, which are both situated immediately on the one surface of A. If in this case $\chi'\ \omega'\ u'$ signifies for the body B, and $\chi''\ \omega''\ u''$ for the body C what $\chi\ \omega\ u$ does for A, we obtain instead of the one conditional equation the two following: —

$$(u)-(u')=a$$
$$(u)-(u'')=a',$$

where a represents the electric tension between the bodies A and B, and a' that between A and C.

In the same manner we now obtain instead of the second conditional equation the following: —

$$\chi\,\omega\left(\frac{d\,u}{d\,x}\right)=\chi'\,\omega'\left(\frac{d\,u'}{d\,x}\right)+\chi''\,\omega''\left(\frac{d\,u''}{d\,x}\right).$$

It is immediately apparent how these equations must change when a greater number of bodies are combined. We shall not enter further into these complications, as what has been stated throws sufficient light upon the changes which have in such a case to be performed on the equations.

14. To avoid misconception, I will, at the close of these observations, once more accurately define the circle of application within which our formulæ have universal validity. Our whole inquiry is confined to the case where all parts of the same section possess equal electroscopic force, and the magnitude of the section varies only from one body to the other.

The nature of the subject, however, frequently gives rise to circumstances which render one or the other of these conditions superfluous, or at least diminishes their importance.

Since the knowledge of such circumstances is not without use, I will here illustrate the most prominent by an example.

A circuit of copper, zinc, and an aqueous fluid will wholly come under the above formula when the copper and zinc are prismatic, and of equal section; when further, the fluid is likewise prismatic, and of the same or of smaller section, and its terminal surfaces everywhere in contact with the metals.

Nay, when only these last conditions are fulfilled with respect to the fluid, the metals may possess equal sections or not, and touch one another with their full sections, or only at some points, and even their form may deviate considerably from the prismatic form, and nevertheless the circuit must constantly obey the laws deduced from our formulæ; for the motion of the electricity produced with such ease in the metals is obstructed to such a considerable extent by the non-conductive nature of the fluid that it gains sufficient time to diffuse itself thoroughly with equal energy over the metals, and thus re-establishes in the fluid the conditions upon which our calculation is founded. But it is a very different matter when the prismatic fluid is only touched in disproportionately small portions of its surfaces by the metals, as the electricity arriving there can only advance slowly and with considerable loss of energy to the untouched surfaces of the fluid, whence currents of various kinds and directions result.

B. *Electroscopic Phenomena.* — 15. In our preceding general determinations we have constantly confined our attention to prismatic bodies, whose axes, upon which the abscissæ have been taken, formed a straight line. But all these considerations still retain their entire value, if we imagine the conductor constantly curved in any way whatsoever, and take the abcissæ on the present curved axis of the conductor.

The above formulæ acquire their entire applicability from this observation, since voltaic circuits, from their very nature, can but seldom be extended in a straight line. Having anticipated this point, we will immediately proceed to the most simple case, where the prismatic conductor is formed in its entire length of the same material, and is curved backwards on itself, and conceive the seat of the electric tension to be where its two ends touch. Although no case in nature resembles this imaginary one, it will nevertheless be of great service in the treatment of the other cases which do really occur in nature.

The electroscopic force, at any point of such a prismatic body, may be deduced from the differential equation (a) found in Sect. 11.

For this purpose we have only to integrate it, and to determine, in accordance with the other conditions of the problem, the arbitrary functions entering into the integral.

The matter is generally much facilitated with respect to our subject by omitting one or even two members, according to the nature of the subject from the equation (a). Thus nearly all voltaic actions are such that the phenomena are permanent and invariable immediately at their origin.

In this case, therefore, the electroscopic force is independent of time, consequently the equation (a) passes into

$$o = \chi \frac{d^2 u}{d x^2} - \frac{b c}{\omega} u .$$

Moreover, the surrounding atmosphere has (as we have seen in Sect. 9) in most cases no influence on the electric nature of the voltaic circuit; then $b = o$, by which the last equation is converted into

$$o = \frac{d^2 u}{d x^2} .$$

But the integral of this last equation is

$$u = fx + c, \qquad (c)$$

where f and c represent any constants remaining to be determined. The equation (c) consequently expresses the law of electrical diffusion, in a homogeneous prismatic conductor, in all cases where the abduction by the air is insensible, and the action no longer varies with time.

As these circumstances in reality are frequent accompaniments of the voltaic circuit, we shall on that account dwell longest upon them. We are enabled to determine one of the constants by the tension occurring at the extremities of the conductor, which has to be regarded as invariable and given in each case. If, for instance, we imagine the origin of the abscissæ anywhere in the axis of the body, and designate the abscissa belonging to one of its ends by x_1, then the electroscopic force there situated is, according to the equation (c),

$$f x_1 + c;$$

in the same way we obtain for the electroscopic force of the other extremity, when we represent its abscissa by x_2,

$$f x_2 + c.$$

If we now call the given tension or difference of the electroscopic force a, we have

$$a = \pm f (x_1 - x_2).$$

But $x_1 - x_2$ evidently represents the entire, positive or negative, length of the prismatic conductor; if we designate this by l, we obtain accordingly

$$a = \pm f\, l,$$

whence the constant f may be determined.

If we now introduce the value of the constant thus found into the equation (c), it is converted into

$$u = \pm \frac{a}{l}\, x + c,$$

so that only the constant c remains to be determined. We may consider the ambiguity of the sign $\pm$ to be owing to the tension a, by ascribing to it a positive value, when the extremity of the conductor, belonging to the greater abscissa possesses the greatest electroscopic force, and when the contrary, a negative.

Under this supposition is then generally,

$$u = \frac{a}{l}\, x + c. \qquad (d)$$

The constant c remains in general wholly undetermined, which admits of our allowing the diffusion of the electricity in the conductor to vary arbitrarily, by external influences, in such manner that it occupies the entire conductor everywhere uniformly.

Among the various considerations respecting this constant, there is one of especial importance to the voltaic circuit; i.e., that which assumes the circuit to be connected at some one place with a perfect conductor, so that the electroscopic force has to be regarded as if it were constantly destroyed at this point. If we call the abscissa belonging to this point, λ, then according to the equation (*d*)

$$o = \frac{a}{l}\lambda + c.$$

By determining from this the constant *c*, and placing its value in the same equation (*d*), we obtain

$$u = \frac{a}{l}(x - \lambda),$$

from which the electroscopic force of a voltaic circuit of the length *l*, and of the tension *a*, which is touched at any given place whose abscissa is λ, may be found for every other point.

If any constant and perfect adduction from outwards to the voltaic circuit were to be given instead of the permanent abduction or abstraction outwardly, so that the electroscopic force pertaining to the abscissa λ were compelled to assume constantly a given energy, which we will designate by α, we should obtain for the determination of the constant c the equation

$$\alpha = \frac{a}{l}\lambda + c,$$

and for the determination of the electroscopic force of the circuit at any other point the following: —

$$u = \frac{a}{l}(\chi - \lambda) + \alpha.$$

We have seen how the constant c may be determined when the electroscopic force is indicated at any place of the circuit by external circumstances; but now the question arises, What value are we to ascribe to the constant when the circuit is left entirely to itself, and this value can consequently no longer be deduced from external circumstances?

The answer is found in the consideration that each time both electricities proceed contemporaneously and in like quantity from a previously indifferent state. It may, therefore, be asserted that a simple circuit of the present kind, which is formed in a perfectly neutral and isolated condition, would assume on each side of the place of contact an equal but opposite electric condition, whence it is self-evident that their centre would be indifferent.

For the same reason, however, it is also apparent that when the circuit at the moment of its origin is compelled by any circumstance to deviate from this its normal state, it would certainly assume the abnormal one, until again caused to change.

The properties of a simple voltaic circuit, such as we have hitherto been considering, accordingly consist essentially in the following, as is directly evident from the equation (*d*):

a. The electroscopic force (potential) of such a circuit varies throughout the whole length of the conductor continually, and on like extents constantly to the same amount; but where the two extremities are in contact, it changes suddenly, and, indeed, from one extremity to the other, to the extent of an entire tension.

b. When any place of the circuit is caused by any circumstance to change its electric state, all the other places of the circuit change theirs at the same time, and to the same amount.

16. We will now imagine a voltaic circuit composed of two parts P and P′, at whose two points of contact a different electric tension occurs, which case includes the thermal circuit. If we call u the electroscopic force of the part P, and u' that of the part P′, then, according to the preceding paragraph, as here, the case there noticed is repeated twice, in consequence of the equation (*c*),

$$u = f x + c$$

for the part P, and

$$u' = f' x + c'$$

for the part P′, where f, c, f', c', are any constant magnitudes to be deduced from the peculiar circumstances of our problem, and each equation is only valid so long as the abscissæ refer to that part to which the equations belong. If we now place the origin of the abscissæ at one of the places of contact of the part P, and suppose the direction of the abscissæ in this part to proceed inwards; moreover, designate by l the length of the part P, and by l' that of P′; and, lastly, represent by u'_2 and u_1 the values of u and u' at the place of contact where $x = l$, we then obtain

$$u'_2 = f'(l + l') + c' \qquad u_1 = c$$
$$u_2 = f\,l + c \qquad u'_1 = f'\,l + c'.$$

If we now designate by a the tension which occurs at the place of contact where $x = o$, and by a' that which occurs at the place of contact where $x = l$; and if we once for all assume, for the sake of uniformity, that the tension at each individual place of contact always expresses the value which is obtained when we deduct the electroscopic force of one extremity from that of the extremity belonging to the place in question, upon which the abscissa falls before the abrupt change takes place — (it is not difficult to perceive that this general rule contains that advanced in the preceding paragraph, and which, in fact, expresses nothing more than that the tensions of such places of contact, by the springing over of which in the direction of the abscissæ we arrive from the greater to the smaller electroscopic force, are to be regarded as positive; in the contrary case as negative; where, however, it must not be overlooked that every positive force has to be taken as greater than every negative, and the negative as greater than the actually smaller), we obtain

$$a = f' \, (l + l') + c' - c,$$

and

$$a' = f \, l - f' \, l + c - c',$$

whence directly results

$$a + a' = f \, l + f' \, l'.$$

But now at each of the places of contact when χ and ω represent the power of conduction and the section of the part P, and

χ' and ω' the same for P′, in accordance with the considerations developed in Sect. 13, there arises the conditional equation

$$\chi\,\omega\left(\frac{d\,u}{d\,x}\right)=\chi'\,\omega'.\left(\frac{d\,u'}{d\,x}\right),$$

where $\left(\frac{d\,u}{d\,x}\right)$ and $\left(\frac{d\,u'}{d\,x}\right)$ represent the values of $\frac{d\,u}{d\,x}$ and $\frac{d\,u'}{d\,x}$ at the place of contact.

From the equations at the commencement of this paragraph for the determination of the electroscopic force in each single part of the circuit, we, however, obtain the value of x to be allowed to each,

$$\frac{d\,u}{d\,x}=f \text{ and } \frac{d\,u'}{d\,x}=f',$$

which converts the conditional equation in question into

$$\chi\,\omega\,f=\chi'\,\omega'\,f'.$$

From this, and the equation $a+a'=$

$f\,l + f'\,l'$ just deduced from the tensions, we now find the values of f and f' thus:—

$$f = \frac{(a + a')\,\chi'\,\omega'}{\chi'\,\omega'\,l + \chi\,\omega\,l'},$$

$$f' = \frac{(a + a')\,\chi\,\omega}{\chi'\,\omega'\,l + \chi\,\omega\,l'},$$

and with the help of these values we find

$$c' = c - a' + \frac{(a + a')\,(\chi'\,\omega'\,l - \chi\,\omega\,l)}{\chi'\,\omega'\,l + \chi\,\omega\,l'}.$$

Hence the electroscopic force of the circuit in the part P is expressed by the equation

$$u = \frac{(a + a')\,\chi'\,a'\,x}{\chi'\,\omega'\,l + \chi\,\omega\,l'} + c,$$

and that in the part P′ by the equation

$$u' = \frac{(a + a')\,\chi\,\omega\,x - \chi\,\omega\,l + \chi'\,\omega'\,l}{\chi'\,\omega'\,l + \chi\,\omega\,l'} - a + c.$$

If we substitute λ and λ' for $\frac{l}{\chi\,\omega}$ and $\frac{l'}{\chi'\,\omega'}$, the following more simple form may be given to these equations:—

$$\left.\begin{aligned} u &= \frac{a+a'}{\lambda+\lambda'} \cdot \frac{x}{\chi\,\omega}\,c \\ u' &= \frac{a+a'}{\lambda+\lambda}\left(\frac{x-l}{\chi'\,\omega'}+\frac{l}{\chi\,\omega}\right) \\ &\quad -a'+c \end{aligned}\right\} \text{(L).}$$

From the form of these equations it will be immediately perceived, that when the conductivity, or the magnitude of the section, is the same in both parts, the expressions for u and u' undergo no other change than that the letter representing the conductivity or the section entirely disappears.

17. We will now proceed to the consideration of a voltaic circuit, composed of three distinct parts P, P′ and P^2, which includes the hydro-circuit.

If we represent by u, u', u'' respectively the electroscopic forces of the parts P, P′ and P″, then, according to Sect. 15, the case there mentioned being here thrice repeated, we have, in accordance with the equation

(c) there found, with respect to the part P,

$$u = f\,x + c',$$

with respect to the part P′,

$$u' = f'\,x + c',$$

and with respect to the part P″,

$$u'' = f''\,x + c'',$$

where f, f', f'', c, c', c'' may represent any constant magnitudes remaining to be determined from the nature of the problem, and each equation has only so long any meaning as the abscissæ refer to that part to which the equations appertain. If we suppose the origin of the abscissæ at that extremity of the part P, which is connected with the part P″, and choose the direction of the abscissæ so that they lead from the part P to that of P′, and from thence into P″; if we further respectively designate by l, l', and l'' the lengths of the parts P, P′, P″; and lastly let u''_2 and u_1 represent the values of u'' and u at the place of contact where $x = o$, and u_2 and u' the values of u and u' at the place of contact where $x = l$, and u'_2 u''_1 the values of u' and u'' at the place of contact where $x = l + l'$, then we obtain

$$u''_2 = f''\,(l + l' + c'') + c''.\ u_1 = c$$
$$u_2 = f\,l + c. \qquad u'_1 = f'\,l + c'$$
$$u'_2 = f'\,(l + l') + c'.\ u''_1 = f''\,(l + l') + c''.$$

If we call a the tension which occurs at the place of contact where $x = o$, a' that at the place of contact where $x = l$, and a'' that at the place of contact where $x = l + l'$, we obtain, if we pay due attention to the general rule stated in the preceding paragraph,

$$a = f''\,(l + l' + l'') + c'' - c$$
$$a' = f\,l - f'\,l' + c - c'$$
$$a'' = f'\,(l + l') - f''\,(l + l') + c' - c'',$$

and hence

$$a + a' + a'' = f\,l + f'\,l' + f''\,l''.$$

But from the considerations developed in Sect. 13 when χ and ω represent the power of conduction and the section for the part P, χ' and ω' the same for the part P′, and

χ'' and ω'' for the part P″, at the respective places of contact, the following conditional, equations are obtained:—

$$\chi\,\omega\left(\frac{d\;u}{d\;x}\right)=\chi'\,\omega'\left(\frac{d\;u'}{d\;x}\right)=\chi''\,\omega''\left(\frac{d\;u''}{d\;x}\right),$$

where $\frac{d\,u}{d\,x}$, $\frac{d\;u'}{d\,x}$, $\frac{d\;u''}{d\;x}$ represent the particular values of $\frac{d\,u}{d\,x}$, $\frac{d\;u'}{d\,x}$, $\frac{d\;u''}{d\,x}$, belonging to the places of contact. From the equations stated at the commencement of the present paragraph for the determination of the electroscopic force in the single parts of the circuit, we obtain for every admissible value of x,

$$\frac{d\,u}{d\,x}=f,\;\frac{d\;u'}{d\;x}=f',\;\frac{d\;u''}{d\;x}=f'',$$

by which the preceding conditional equations are converted into

$$\chi\,\omega\,f=\chi'\,\omega'\,f'=\chi''\,\omega''\,f''.$$

From these, and the equation between f, f' f'' above deduced from the tensions, we now find, when λ, λ' λ'', are respectively substituted for

$$\frac{l}{\chi\,\omega},\ \frac{l'}{\chi'\,\omega'},\ \frac{l''}{\chi''\,\omega''},$$

$$f = \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\chi\,\omega},$$

$$f' = \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\chi'\,\omega'},$$

$$f'' = \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\chi''\,\omega''},$$

and by the aid of these values we find further

$$c' = \frac{a + a' + a''}{\lambda + \lambda' + \lambda''}\left(\frac{l}{\chi\,\omega} - \frac{l}{\chi'\,\omega'}\right) - a' + c,$$

$$c'' = \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{l}{\chi'\,\omega'} - \frac{l + l'}{\chi''\,\omega''} + \frac{l}{\chi\,\omega}\right) - (a' + a'') + c.$$

By substituting these values, we obtain for the determination of the electroscopic force of the circuit in the parts P, P′, P″ respectively, the following equations:

$$\left.\begin{aligned}
u &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{x}{\chi\,\omega} + c \\
u' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{x - l}{\chi'\,\omega'} + \frac{l}{\chi\,\omega}\right) \\
&\qquad - a' + c \\
u'' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{x - (l + l')}{\chi''\,\omega''} + \right. \\
&\left. \frac{l'}{\chi'\,\omega'} + \frac{l}{\chi\,\omega}\right) - (a' + a'') + c
\end{aligned}\right\} \text{(L}'\text{)}$$

and it is easy to see that these equations with the omission of the letter χ or ω (both where they are explicit, as well as in the expressions for λ, λ' λ''), are the true ones for the case $\chi = \chi'$, or $\omega = \omega' = \omega''$.

18. These few cases suffice to demonstrate the law of progression of the formulæ ascertained for the electroscopic force, and to comprise them all in a single general expression.

To do this with the requisite brevity, for the sake of a more easy and general survey, we will call the quotients, formed by dividing the length of any homogeneous part of the circuit by its power of conduction and its section, *the reduced length* of this part; and when the entire circuit comes under consideration, or a portion of it composed of several homogeneous parts, we understand by its reduced length the sum of the reduced lengths of all its parts.

["Thus the reduced length of a circuit is *the length of a wire, of a given nature and thickness, whose resistance is equal to the sum of the resistances of this circuit.* It is a very convenient mode of expressing the resistance to conductability, presented by the whole or by a part of a circuit, to reduce it to that which would be presented by a certain length of wire of a given nature and diameter." — *Treatise on Electricity.* De La Rive. Vol. ii. 1856, pp. 80, 81.]

Having premised this, all the previously found expressions for the electroscopic force, which are given by the equations

(L) and (L′), may be comprised in the following general statement, which is true when the circuit consists of any number of parts whatever.

The electroscopic force of any place of a galvanic circuit, composed of any number of parts, is found by dividing the sum of all its tensions by its reduced length, multiplying this quotient by the reduced length of the part of the circuit comprised by the abscissa, and subtracting from this product the sum of all the tensions abruptly passed over by the abscissa; lastly, by varying the value thus obtained by a constant magnitude to be determined elsewhere.

If therefore we designate by A the sum of all the tensions of the circuit, by L its entire reduced length, by y the reduced length of the part which the abscissa passes through, and by O the sum of all the tensions to the points to which the abscissa corresponds, lastly by u, the electroscopic force of any place in any part of the circuit, then

$$u - \frac{\mathrm{A}}{\mathrm{L}} y - \mathrm{O} + c,$$

where c represents a constant, but yet undetermined magnitude.

Thus transformed, this exceedingly simple expression for the electroscopic force of any circuit will allow us hereafter to combine generality with conciseness, for which purpose we will moreover indicate by y the reduced abscissa. This form of the equation has besides the peculiar advantage that without anything further, it is even applicable when in any part of the circuit the tensions and conductivities constantly vary; for, in this case, instead of the sums we should merely have to take the corresponding integrals, and to define their limits according as the nature of the expression required.

Since O does not change its value within the entire extent of the same homogeneous part of the circuit, and y constantly varies to the same amount on like portions of this extent, the following properties, already demonstrated less generally with respect to the simple circuit, evidently apply to every voltaic circuit, and in these is expressed the main character of voltaic circuits: —

a. The electric force of each homogeneous portion of the circuit varies throughout its entire length constantly, and on like extents always to the same amount; but where it ceases and another commences, it suddenly changes to the extent of the entire tension situated at that place.

b. If any single place of the circuit is induced by any circumstance whatsoever to change its electric condition, all the other places of the circuit change theirs at the same time, and to the same amount.

The constant c is in the rule determined by ascertaining the electroscopic force at any point of the circuit. If, for instance, we designate by u' the electroscopic force at a place of the circuit, the reduced abscissa of which is y', then in accordance with the general equation above stated,

$$u' = \frac{A}{L} y' - O' + c,$$

where O' represents the sum of the tensions abruptly passed over by the abscissa y'.

If we now subtract this equation, valid

for a certain place of the circuit, from the previous one belonging in the same manner to all places, we obtain

$$u - u' = \frac{A}{L}(y - y') - (O - O'),$$

in which nothing more remains to be determined.

If the circuit, during its production, is not exposed to any external deduction or adduction, the constant c must be sought for in the circumstance that the sum of all the electricity in the circuit must be zero.

This determination is founded on the fundamental position, that from a previously indifferent state, both electricities constantly originate at the same time and in like quantity.

To illustrate by an example, the mode in which the constant c is found in such a case, we will again consider the case treated of in Sect. 16. In the portion P of that circuit, u is generally $= \frac{A}{L} y + c$,

where $y = \frac{x}{\chi \omega}$, and in the portion P′ we have constantly $u = \frac{A}{L} y - a' + c$, where

$y = \frac{x - l}{\chi' \omega'} + \lambda$.

Since now the magnitude of the element, in the portion P, is $\omega \, d \, x$, or $\chi \, \omega^2 \, d \, y$, but in the portion P′ is $\omega' \, d \, x$ or $\chi' \, \omega'^2 \, d \, y$, we obtain for the quantity of electricity contained in an element of the first portion

$$\chi \, \omega^2 \, d \, y \left(\frac{A}{L} y + c \right),$$

and for the quantity contained in an element of the second portion

$$\chi' \, \omega'^2 \, d \, y \left(\frac{A}{L} y - a' + c \right).$$

If we now integrate the first of the two preceding expressions from $y = 0$ to $y = \lambda$, we then obtain for the whole quantity of electricity contained in the part P,

$$\chi \, \omega^2 \left[\frac{A}{2 \, L} \lambda^2 + c \, \lambda \right];$$

in the same manner we obtain, by integrating the second expression from $y = \lambda$, to $y = \lambda + \lambda'$, for the entire quantity of electricity contained in the portion P'

$$\chi' \omega'^2 \left[\frac{A}{2\,L} (\lambda'^2 + 2\,\lambda\,\lambda') - a'\,\lambda' + c\,\lambda' \right]$$

But the sum of the two last found quantities must, in accordance with the above-advanced fundamental position, be zero.

We thus obtain the equation required for the determination of the constant c, and it only remains to be observed that λ and λ' are the reduced lengths corresponding to the portions P and P'.

We have hitherto tacitly assumed only positive abscissæ. But it is easy to show that negative abscissæ can be quite as well introduced.

For let $-y$ represent such a negative reduced abscissa for any point in the circuit, then $L - y$ is the positive reduced abscissa pertaining to the same place, for which the general equation is found valid; we accordingly obtain

$$u = \frac{A}{L} (L - y) - O + c$$

or

$$u = - \frac{A}{L} y - (O - A) + c.$$

But O — A evidently expresses, if regard be had to the general rule expressed in Sect. 16, the sum of all the tensions abruptly passed over by the negative abscissa, whence it is evident that the equation still retains entire its former signification for negative abscissæ.

19. If we imagine one of the parts of which the voltaic circuit is composed, to be a non-conductor of electricity, i.e., a body whose capacity of conduction is zero, the reduced length of the entire circuit acquires an indefinitely great value.

If we now make it a rule never to let the abscissæ enter into the non-conducting part, in order that the reduced abscissæ y may constantly retain a finite value, the general equation changes into the following:

$$u = - O + c,$$

which indicates that the electroscopic force in the whole extent of each other homogeneous portion of the circuit is everywhere the same, and merely changes suddenly from one part to the other to the amount of the entire tension prevailing at its place of contact.

To determine the constant c in this equation, we will suppose the electroscopic force at any one place of the circuit to be given. If we call this u', and the sum of the tensions there abruptly passed over by the abscissa O′, we have

$$u - u' = -(O - O').$$

The difference of the electroscopic forces of any two places of an open circuit, i.e., a voltaic circuit interrupted by a non-conductor, is consequently equal to the sum of all the tensions situated between the two places, and the sign which has to be placed before this sum is easily to be determined from mere inspection.

20. We will now notice another peculiarity of the voltaic circuit which merits special attention. To this end let us keep in view exclusively one of the homogeneous parts of the circuit, and imagine, for the sake of simplicity, the origin of the abscissæ placed in one end of it, and the abscissæ directed towards the other end.

If we designate its reduced length by λ, and the reduced length of the other portion of the circuit by $\varLambda$, then

$$u = \frac{\mathrm{A}}{\varLambda - \lambda} \cdot y + c$$

within the length λ; the following form may also be given to this equation:

$$u = \frac{\frac{\mathrm{A}\,\lambda}{\varLambda + \lambda}}{\lambda} \cdot y + c;$$

the extent is consequently similarly circumstanced to a simple homogeneous circuit, at whose ends the tension $\frac{\mathrm{A}\,\lambda}{\varLambda + \lambda}$ occurs. If, accordingly, A has a very sensible value, such as it can acquire in the voltaic battery, and if the ratio $\frac{\lambda}{\varLambda + \lambda}$ approaches to unity, then the tension $\frac{\mathrm{A}\,\lambda}{\varLambda + \lambda}$ will likewise be still very perceptible; consequently its various gradations in the extent of the portion λ are very easily perceptible.

This conclusion is of importance, because it affords the means of presenting to the senses the law of electric distribution even on compound circuits, when it is no longer possible on the simple circuit on account of its extremely feeble force. It is, moreover, immediately evident, that with equal tensions this phenomenon will be indicated with greater intensity, the greater λ is, in comparison with $\varLambda$.

21. A phenomenon common to all voltaic circuits is the sudden change to which its electroscopic force may incessantly and arbitrarily be subjected. This phenomenon has its source in the previously developed properties of such circuits. Since, as we have found, each point of a voltaic circuit undergoes the same alterations to which a single one is exposed, we have it in our power to give sometimes one, sometimes another, value to the electroscopic force at any certain place. Among these changes those are the most remarkable which we are able to produce by making contact with a body at zero potential, i.e., by destroying the electroscopic force or bringing it to a value of zero sometimes at one, and sometimes at another point of the cir-

cuit; its magnitude, however, has its natural limits in the magnitude of the tensions.

There is another class of phenomena which is immediately connected with these. If, for instance, we call the space over which the electroscopic force is diffused in a given voltaic circuit r, call u the electroscopic force of the circuit at one of its points, which is immediately connected with an external body M, and apply the symbol u' to the electroscopic force of the same circuit at the same place as it was before making contact with the body M, $u' - u$ is evidently the alteration in the electroscopic force produced at this place; consequently, since this change likewise occurs uniformly at all the other places of the circuit, $r\ (u' - u)$ is the quantity of electricity which the change produced over the entire circuit comprises, and accordingly that which has passed over into the body M.

If now we suppose that in the state of equilibrium the electroscopic force is everywhere of equal intensity at all places of the body M in which it occurs, and represent by R the space over which it is diffused in the body M, then its electroscopic force is evidently $\frac{r\ (u' - u}{\mathrm{R}}$

But this force is in the state of equilibrium equal to the u', which the place of the circuit, brought into contact with M, has assumed when no new tension originates at this place of contact; under this supposition, therefore, $u = \frac{r\ (u' - u}{\mathrm{R}}$, whence we find $u = \frac{r\ u'}{r + \mathrm{R}}$.

From this equation it results that the electroscopic force in the body M will constantly be smaller than it was at the zero touched point before contact; and also that both will approximate the more to each other, the greater r is in comparison with R. If we regard R as a constant magnitude, the relation of the electroscopic forces u and u' to each other depends solely upon the magnitude of the space which the electricity occupies in the circuit: we can therefore bring the electroscopic force of the body M nearer to its greatest value solely by increasing the capacity of the circuit, either by a general increase of its dimensions, or by attaching anywhere to it foreign masses.

None of this effect seems to depend upon the nature of these masses, when they are merely conductors of electricity, and do not give rise to new tensions, but solely upon their magnitudes.

If the attached masses occupy an infinitely great space, which case occurs when the circuit has anywhere a complete deduction, then the electroscopic force in the body M will constantly be equal to that which the place of the circuit touched by it possesses.

To connect these effects with the action of the condenser, we have merely to bear in mind that a condenser whose capacity is R, and whose number of charges is m, must be considered equal to a common conductor of the magnitude m R, yet with the difference that its electroscopic force is m times that of the common conductor. If, therefore, we designate by u the electroscopic force of the condenser, which is brought into connection with a place of the circuit whose force is u' we obtain

$$u = \frac{m\, r\, u'}{r + m\, R},$$

whence it follows that the condenser will indicate m times the force of the touched place when r is very great in comparison with m R; but that it will have a weakening action so soon as r is equal to, or smaller than R.

Masses attached anywhere to the circuit will accordingly make the indications of the condenser approximate to its maximum in proportion as they are greater, and a circuit touched at any place will constantly produce in the condenser the maximum of increase.

The preceding determinations suppose that one plate of the condenser remains constantly touched deductively. We will now take into consideration the case where the two plates of an insulated condenser are connected with various points of a voltaic circuit.

In the first place, it is evident that the two plates of an insulated condenser will assume the same difference of free electricity which the various places of the circuit with which they are in contact require unconditionally, from the peculiar nature of voltaic actions. Consequently if d represents the difference of the electroscopic force at the two points of the circuit, and u the free electricity of one plate of the condenser, then $u + d$ is the free electricity of the other plate, and everything will depend on finding, from the known free electricities existing in the plates of the condenser, those actually present in them. If, for this purpose, we call A the actual intensity of electricity in the plate, whose free electricity is $u + d$, then $A - u - d$ represents the portion of electricity retained in the same plate; in the same manner $B - u$ designates the portion of electricity retained in the plate whose free electricity is u, when B represents the actual intensity of the electricity in this plate.

If now we represent by n the relation

between the electricity retained by one plate, and the actual electricity of the other plate, the following two equations arise:

$$A - u - d + n\,B = o,$$
$$B - u + n\,A = O,$$

from which the values A′ and B result as follows:

$$A = \frac{d + u\ (1 - n)}{1 - n^2},$$

$$B = u\ \frac{(1 - n) - n\,d}{1 - n^2},$$

But from the theory of the condenser, it is well known that $1 - n = \frac{1}{m}$, if m is the number of charges of the condenser; if, therefore, we substitute $\frac{1}{m}$ for $1 - n^2$ in the expressions for A and B, and at the same time $1 - \frac{1}{2\ m}$ for n, which is permitted when m, as is usually the case, denotes a very large number, we obtain

$$A = m\,d + \tfrac{1}{2}\,u,$$
$$B = -\,m\,d + \tfrac{1}{2}\,u + \tfrac{1}{2}\,d.$$

Or when m is a very large number, and n not much larger than d, we may, without committing any perceptible error, place

$$A = m\,d,$$
$$B = -\,m\,d,$$

in which is expressed the known law, that when two different points of a voltaic battery are brought into connection with the two plates of an insulated condenser each plate takes the same charge as if the other plate, and the corresponding place of the battery, had been touched deductively. At the same time our considerations show that this law ceases to be true when u can no longer be regarded as evanescent towards $m\ d$.

This case would occur if, for instance, two points near the insulated upper pole of a voltaic pile, constructed of a great number of elements, came in contact with the plates of the condenser, while the lower pole of this pile remained in connection with the earth, thus maintaining a zero potential.

The determination hitherto given respecting the mode in which the voltaic circuit imparts its electricity to foreign bodies, might, however, give rise to researches of a very different kind, and of no slight interest.

For it is placed beyond all doubt, both theoretically and experimentally, that electricity in motion penetrates into the interior of bodies, and its quantity accordingly depends on the space occupied by the bodies; while it is equally well ascertained that static electricity accumulates at the surface of bodies, and its quantity therefore is dependent on the extent of surface. But it would hence result that in the closed voltaic circuit, r in the above formulæ would express the volume of the circuit; in the open circuit, on the contrary, the magnitude of its surface, on which point, in my opinion, experiments might decide without great difficulty.

22. We have hitherto kept in view a circuit on which the surrounding atmosphere exercised no influence, and which has already arrived at its permanent

state, and we have treated it at a length which it merits from the abundance and importance of the phenomena connected with it. However, that the other circuits may not pass entirely unnoticed, we will briefly indicate the method to be pursued for the most simple case, and thus point out the path to be followed, although only at a distance.

If it is intended to take into consideration the influence of the atmosphere on the galvanic circuit, the member $\frac{b\,c}{\omega}\,u$ must be added to the member $\chi\,\frac{d^2\,u}{d\,x^2}$ of the equation (a) in Sect. 11, we then obtain for the circuit which has acquired a permanent state, for which $\frac{d\,u}{d\,t} = 0$, the equation

$$0 = \chi\,\frac{d^2\,u}{d\,x^2} - \frac{\lambda\,c}{\omega}\,u\;;$$

or, if we put $\frac{\lambda}{\chi}\,\frac{c}{\omega} = \beta^2$,

$$0 = \frac{d^2\,u}{d\,x^2} - \beta^2\,u.$$

The integral of this equation is

$$u = c \,.\, e^{\beta x} + d \,.\, e^{-\beta x},$$

where e represents the base of the natural logarithms, and c, d, any constants to be determined from the other circumstances of the problem. If we now call $2\ l$ the length of the entire circuit, and fix the origin of the abscissæ in that place of the circuit which is equidistant from the point of excitation; if, further, we designate by a the tension existing at the point of excitation, we obtain

$$a = (c - d)\ (e^{\beta l} - e^{-\beta l}).$$

If we write the previously found equation thus,

$$u = (c - d)\ e^{\beta x} + d\ (e^{\beta x} + e^{-\beta x}),$$

and substitute for $c - d$, the value just ascertained, we have

$$u = \frac{a \,.\, e^{\beta x}}{e^{\beta l} - e^{-\beta l}} + d\ (e^{\beta x} + e^{-\beta x}).$$

If we now suppose for the determination of the other constant, that the sum of the two electroscopic forces, situated at the point of excitation, is known, and is equal to b, which case always occurs when the electroscopic force of the circuit is given at any one of its places, we obtain

$$b = a\,\frac{(e^{\beta l} + e^{-\beta l})}{e^{\beta l} - e^{-\beta l}} + 2\,d\,(e^{\beta l} + e^{-\beta l});$$

and after substitution and proper reduction,

$$u = \frac{\frac{1}{2}\,a\,(e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}} + \frac{\frac{1}{2}\,b\,(e^{\beta x} + e^{-\beta x})}{e^{\beta l} + e^{-\beta l}},$$

which for $b = 0$, i.e., for a circuit left entirely to itself, changes into

$$u = \frac{\frac{1}{2}\,a\,(e^{\beta x} - e^{-\beta x}}{e^{\beta l} - e^{-\beta l}}.$$

These equations which hold for a circuit homogeneous and prismatic in its whole extent, change when $\beta = 0$ again into the above, where the influence of the atmosphere on the circuit was, under the circumstances given above, left out of consideration.

Since $\beta^2 = \frac{b}{\chi} \cdot \frac{c}{\omega}$, it follows that the influence of the atmosphere on the voltaic circuit must be less, the smaller the conducting power of the atmosphere is in comparison to that of the circuit, and the smaller the quotient $\frac{c}{\omega}$ is. But the quotient $\frac{c}{\omega}$ expresses the relation of the surface of a disk of the conductor surrounded by the atmosphere to the volume of the same disk, and it might therefore appear that $\frac{c}{\omega}$ must constantly be infinitely small.

However, we are not here dealing with mathematical but with physical determinations; for strictly taken, c does not represent a surface, but that portion of a disk of a circuit on which the atmosphere has direct influence, and ω in fact signifies nothing more than that part of a disk of the circuit which is traversed by the electricity continually passing through the circuit.

In general, therefore, c is indeed incomparably smaller than ω; but where the electric current can only move forwards with the greatest difficulty, and on that account very slowly, as is more or less the case in dry piles, the magnitude c may, in accordance with what was stated in the preceding paragraph, become very nearly equal to ω; for undoubtedly a gradual transition. modified by the contemporaneous circumstances, must occur from that which is peculiar to the rapid current, to that belonging to the state of perfect equilibrium.

Here, then, is a wide field open for future researches.

23. In cases where the permanent state is not instantaneously assumed, as it usually is in dry piles, we should in order to become acquainted with the changes of the circuit up to that period, proceed from the complete equation

$$\gamma \frac{d u}{d t} = \chi \frac{d^2 u}{d x^2} - \frac{b c}{\omega} u, \qquad (*)$$

because in this case we cannot consider $\frac{d u}{d t} = 0$, and the member $\frac{b c}{\omega} u$ must either remain in it, or be removed from it, according to whether it is considered worth while to take the influence of the atmosphere on the circuit into consideration or not.

If we again place, as in the previous paragraph, $\beta^2 = \frac{b\,c}{\chi\,\omega}$, and, also $\frac{\chi}{\gamma} = \chi'$, the preceding equation changes into the following:

$$\frac{d\,u}{d\,t} = \chi' \left(\frac{d^2\,u}{d\,x^2} - \beta^2\,u\right),$$

and we immediately perceive, that on admitting $\beta = 0$, the action of the atmosphere is left out of the question.

In the present case u represents a function of x and t, which, however, in proportion as the time t increases, becomes gradually less dependent on t, and at last passes over into a mere function of x, which expresses the permanent state of the circuit, with the nature of which we have already become acquainted.

If we designate this latter function by u', and place $u = u' + v$, then v is evidently a function of x and t, which indicates every deviation of the circuit from its permanent state, and consequently after the lapse of a certain time entirely disappears.

If we now substitute $u' + v$ for u in the equation (*), and bear in mind that u' is independent of t, and of such nature that

$$0 = \frac{d^2 u'}{d\, x^2} - \beta^2 u',$$

the equation

$$\frac{d\, y}{d\, t} = \chi' \left(\frac{d^2 v}{d\, x^2} - \beta^2 v \right) \qquad (☽)$$

then remains for the determination of the function v, which still possesses the same form as the equation (*), but differs from it in this respect, that v is a function of x, and t of a different nature from u, by which its final determination is much facilitated.

The integral of the equation (☽), in the form in which it was first obtained by Laplace, is

$$v = \frac{e^{-\chi' \beta^2 t}}{\sqrt{\pi}} \int e^{-\nu^2} f(x + {}^2 y \sqrt{\chi' t})$$

$$d\, y, \qquad (☿)$$

where e represents the base of the natural logarithms, π the ratio of the circumference of a circle to its diameter, and f an arbitrary function to be determined from the peculiar nature of each problem, while the limits of the integration must be taken from $y = -\infty$ to $y = +\infty$. For $t = 0$ we have $v = f\,x$, because between the indicated limits $\int e^{-y^2} d\,y = \sqrt{\pi}$, whence it results that if we know how to find the function v in the case where $f = 0$, we should thereby likewise discover $f\,x$, consequently the arbitrary function f.

Now in general $v = u - u'$; but if we reckon the time t from the moment when, by the contact at the two extremities of the circuit, the tension originates, then, u when $t = 0$ has evidently fixed values only at these extremities, at all other places of the circuit u is $= 0$; accordingly, in the whole extent of the circuit $v = - u'$ in general when $t = 0$; only at the extremities of the circuit at the same time $v = u - u'$.

If, therefore, we imagine a circuit left from the first moment of contact entirely to itself, then v constantly $= 0$ at its extremities, so that therefore in the interior of the circuit $v = -u'$, when $t = 0$, and at its ends $v = 0$.

Since in accordance with our previous inquiries, u' may be regarded as known for each place of the circuit, this likewise applies to v when $t = 0$; we know then the form of the arbitrary function $f\,x$, so long as x belongs to a point in the circuit.

However, the integral given for the determination of v requires the knowledge of the function $f\,x$ for all positive and negative values of x; we are thus compelled to give, by transformation, such as the researches respecting the diffusion of heat have made us acquainted with, such a form to the above equation that only presupposes the knowledge of the function $f\,x$ for the extent of the circuit.

The transformation applicable to the present case gives, when $2\,l$ designates the length of the circuit, and the origin of the abscissæ is placed in its centre,

$$v = \frac{e^{-\chi^1 \beta^2 t}}{l} \Big[\Sigma \Big(e^{\frac{-\chi^1 i^2 \pi^2 t}{l^2}} \cdot \sin \frac{i \pi x}{l} \int \sin \frac{i \pi y}{l} f y \, d y \Big) + \Sigma$$

$$\Big(e^{\frac{-(2i-1)^2 \pi^2 t}{4 l^2}} \cos \frac{(2i-1) \pi \chi}{2 l} \int \cos \frac{(2i-1) \pi y}{2 l} f \dot{y} \, d y \Big) \Big]$$

where the sums must be taken from $i = 1$ to $i = \infty$, and the integrals from $y = -l$ to $y = +l$. If we now substitute in this equation for $f\,x$ its value $-u'$, whereby according to our supposition in the preceding paragraph, if a represents the tension at the place of contact,

$$u^1 = \frac{\frac{1}{2}\, a\, (e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}},$$

and then integrate, we obtain, since between the indicated limits

$$\tfrac{1}{2}\, a \int \sin \frac{i \pi y}{l} \cdot \frac{e^{\beta v} - e^{-\beta v}}{e^{\beta l} - e^{-\beta l}} \cdot d y =$$

$$-\frac{a\, i\, \pi\, l \cos i \pi}{i^2 \pi^2 + \beta^2 l^2},$$

and

$$\tfrac{1}{2} a \int \frac{e^{\beta y} - e^{-\beta y}}{e^{k l} - e^{-k l}} \cdot \cos \frac{(2 i - 1 \pi y}{2 l} . \, d y = 0,$$

for the determination of v the equation

$$v = a \, . \, e^{-\chi^1 \beta^2 t} \, \Sigma \left(\frac{i \pi \sin \frac{i \pi (l + x}{l}}{i^2 \pi^2 + \beta^2 l^2} . \, e^{\frac{- \chi^1 \pi^2 i^2 t}{l^2}} \right),$$

and lastly, since $u = u' + v$

$$u = \frac{\frac{1}{2} a (e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}} + a \, . \, e^{-\chi^1 \beta^2 t} \times \Sigma \left(\frac{i \pi \sin \frac{i \pi (l + x)}{l}}{i^2 \pi^2 + \beta^2 l^2} . \, e^{\frac{- \chi^1 \pi^2 i^2 t}{l^2}} \right),$$

which equation, for $\beta = 0$, i.e., when it is not intended to take into consideration the influence of the atmosphere, passes into

$$u = \frac{u}{2 l} x + a \, \Sigma \left(\frac{1}{i \pi} \sin \frac{i \pi (l + x)}{l} . \, e^{\frac{- \chi^1 \pi^2 i^2 t}{l^2}} \right).$$

It is easily perceived that the value of the second member to the right in the equations which have been found for the determination of u becomes smaller and smaller as the time increases, and that it at last entirely vanishes; the permanent state of the circuit has then occurred.

This moment can, as is evident from the form of the expression, be retarded by a diminished power of conduction, and in a far greater degree by an increased length of the circuit.

This expression found for u, however, holds perfectly only so long as the circuit, which we have supposed, is not induced by any external disturbance to change its natural state. If the circuit is at any time compelled by any external cause (for instance by deductive contact at any place) to approximate to an altered permanent state, the above method has to experience some changes which are intended to be developed on another occasion.

It is to be stated that it is in this last class of voltaic circuits, where the peculiar phenomena of dry piles, and in general also of unusually long circuits, are to be sought (and to this class belong the circuits of very great length employed in the experiments of Basse, Erman, and Aldini), provided in the latter case, that the influence of their great length be not annulled by an improved conductivity, or by an enlarged section.

C. *Phenomena of the Electric Current.* — 24. According to what was advanced in paragraph 12, the magnitude of the electric current, in a prismatic body, will generally be expressed for each of its places by the equation

$$S = \omega \, \chi \frac{du}{dx},$$

where S denotes the strength of the current, and u the electroscopic force at that place of the circuit whose abscissa is x, while ω represents the section of the prismatic body, and χ its power of conduction at the same place. To connect this equation with the general equation found in Sect. 18 for any circuit, composed of any number of parts, we write it thus:

$$S = \chi\,\omega\,\frac{d\,u}{d\,y}\cdot\frac{d\,y}{d\,x},$$

and substitute for $\frac{d\,u}{d\,y}$ the value $\frac{A}{L}$ resulting from that general equation, and for $\frac{d\,y}{d\,x}$ the value $\frac{1}{\chi\,\omega}$ easily deducible from the same paragraph, both which values are valid for each place situated between two points of excitation, and we then obtain the extremely simple equation $S = \frac{A}{L}$, where L denotes the entire resistance of the circuit, and A the sum of all its electromotive forces.

["The fundamental expression is $C\ \frac{E}{R}$ or $\frac{\text{force}}{\text{resistance}} = \text{current}$. It follows that any two of the three elements, E R and C, being known, we can calculate the third thus:

Current. $\frac{E}{R} = C$. Force and resistance being known.

Electromotive force. $C \times R = E$. Current and resistance being known.

Resistance. $\frac{E}{C} = R$. Thus, with any source the electromotive force of which is known (in volts), dividing this by the current in amperes, gives us the total resistance of the circuit." — *Electricity.* Sprague. 1st edition, 1875, pp. 199, 200.]

By means of this equation, we obtain the strength of the electric current of a voltaic circuit, composed of any number of prismatic parts, which has acquired its permanent state, which is not affected by the surrounding atmosphere, and the single sections of which possess in all their points one and the same electroscopic force; in this category are comprised the most frequently occurring cases, on which account we shall dissect this result in the most careful manner.

Since A represents the sum of all the tensions in the circuit, and L the sum of the reduced lengths (the resistances) of all the individual parts, there results, in the first place, from the equation found, the following general properties relative to the electric current of the voltaic circuit.

A. The electric current is decidedly of equal strength at all places of a voltaic circuit, and is independent of the value of the constant *c*, which, as we have seen, fixes the intensity of the electroscopic force at a determined place. In the open circuit the current ceases entirely, for in this case the reduced length L acquires an infinitely great value.

B. The strength of the current, in a voltaic circuit, remains unchanged when the sum of all its tensions and its entire reduced length are varied either not at all or in the same proportion; but it increases, the reduced length remaining the same, in proportion as the sum of the tensions increases, and the sum of the tensions remaining the same, in proportion as the reduced length of the circuit diminishes. From this general law we will particularly deduce the following:

(1) A difference in the arrangement and

distribution of the individual points of excitation, by transposing the parts of which the circuit consists, has no influence on the strength of the current, when the sum of all the tensions remains the same. Thus, for instance, the current would remain unaltered in a circuit formed in the order copper, silver, lead, zinc, and a fluid, even when the silver and lead change places with each other; because, according to the laws of tension observed with respect to metals, this transposition would, it is true, alter the individual tensions, but not their sum.

(2) The strength of a voltaic current continues the same, although a part of the circuit be removed, and another prismatic conductor be substituted in its place, only both must have the same reduced length, and the sum of the tensions in both cases remains the same; and *vice versa*, when the current of a circuit is not altered by the substitution of one of its parts for a foreign prismatic conductor, and we can be convinced that the sum of the tensions has remained the same, then the reduced

lengths of the two exchanged conductors are equal.

[“The result of these investigations in the case of homogeneous conductors is commonly called ‘Ohm’s Law.’

Ohm’s Law.

The electromotive force acting between the extremities of any part of a circuit is the product of the strength of the current, and the resistance of that part of the circuit.

Here a new term is introduced, the resistance of a conductor, which is defined to be the ratio of the electromotive force to the strength of the current which it produces. The introduction of this term would have been of no scientific value unless Ohm had shown, as he did experimentally, that it corresponds to a real physical quantity; that is, that it has a definite value which is altered only when the nature of the conductor is altered.

In the first place, then, the resistance of the conductor is independent of the strength of the current flowing through it.

In the second place the resistance is independent of the electrical potential at which the conductor is maintained, and of the density of the distribution of electricity on the surface of the conductor.

It depends entirely on the nature of the material of which the conductor is composed, the state of aggregation of its parts, and its temperature.

The resistance of a conductor may be measured to within one ten thousandth, or even one hundred thousandth part of its value, and so many conductors have been tested that our assurance of the truth of Ohm's Law is now very high." — *Electricity and Magnetism.* MAXWELL. 1873. Vol. i. pp. 296, 297.]

(3) If we imagine a galvanic circuit always constructed of a like number of parts, of the same substance, and arranged in the same order, to the end that the individual tensions may be regarded as unchangeable, the current of this circuit

increases, the length of its parts remaining unaltered, in the same proportion in which the sections of all its parts increase in a similar manner, and the sections remaining unaltered, in the same proportion in which the length of all its parts uniformly decrease.

When the reduced length of a part of the circuit far exceeds that of the other parts, the strength of the current will mainly depend on the dimensions of this part; and the law here announced will assume a much simpler form, if, in the comparison, attention be solely directed to this one part.

The conclusion reached in B. (2) presents a convenient means for the determination of the conductivity of various bodies.

If, for instance, we imagine two prismatic bodies, whose lengths are l and l', their sections respectively ω and ω', and whose powers of conduction are χ and χ', and both bodies possess the property of keeping the current of a voltaic circuit at the same strength, when they alternatively form a part of the said circuit; and both leave the individual tensions of the circuit unchanged, then

$$\frac{l}{\chi\,\omega} = \frac{l'}{\chi'\omega'},$$

consequently

$$\chi : \chi' = \frac{l}{\omega} : \frac{l'}{\omega'};$$

the powers of conduction, therefore, of both bodies are directly proportionate to their lengths, and inversely proportionate to their sections.

If it is intended to employ this relation in the determination of the conducting powers of various bodies, and we choose for the experiments prismatic bodies of the same section, which indeed is requisite for the sake of accuracy, their lengths will enable us to determine accurately their conductivities.

25. In the preceding paragraph we have deduced the current strength from the general equation given in Sect. 18,

$$u = \frac{A}{L}\,y - O + c,$$

and have found that it is expressed by $\frac{A}{L}$, the coefficient of y.

To ascertain the value $\frac{A}{L}$ it is in general necessary to possess an accurate knowledge of all the component parts of the circuit, and their reciprocal tensions; but our general equation indicates a means of deducing this value likewise from the nature of any single part of the circuit, in the state of action, which we will not disregard, as we shall find it serviceable hereafter.

If, namely, we conceive in the above equation y to be increased by any magnitude $\Delta\, y$, and designate by Δ O the corresponding change of O, and by A u that of u, there results from that equation

$$\Delta\, u = \frac{A}{L}\Delta O - \Delta\, O,$$

and we thence find

$$\frac{A}{L} = \frac{\Delta\, u + \Delta\, O}{\Delta\, y};$$

we find, therefore, the magnitude of the electric current by adding to the difference of the electroscopic forces at any two places of the circuit, the sum of all the tensions situated between these two places, and dividing this sum by the reduced length of the part of the circuit which lies between these same places.

If there should be no tension within this portion of the circuit, then $\varDelta\ O = 0$, and we obtain

$$\frac{A}{L} = \frac{\varDelta\ u}{\varDelta\ y}.$$

26. The voltaic battery which is a combination of several similar simple circuits merits peculiar attention at this point from the numerous and varied experimental results obtained by its means.

If A represents the sum of the tensions of a closed voltaic circuit, and L its reduced length, the magnitude of its current is, as we have found,

$$\frac{A}{L}.$$

Now, if we imagine n such circuits perfectly similar to the former, but open; and constantly bring the end of each one into direct connection with the beginning of the next following one, so that between each two circuits no new tension occurs, and all the previous tensions remain afterwards as before, then the strength of the current of this voltaic combination closed in itself, is evidently

$$\frac{n\,A}{n\,L},$$

and consequently equal to that of the simple circuit.

This equality of the circuit, however, no longer exists when a new conductor, which we will call the interposed conductor, is inserted in both.

If, namely, we designate the reduced length of this interposed conductor by $\varLambda$, then, when no new tension is produced by it, the strength of the current in the simple circuit will be $\frac{A}{L + \varLambda}$, and in the voltaic combination consisting of n, such elements $\frac{n\,A}{n\,L + \varLambda}$ or $\frac{A}{L + \frac{\varLambda}{n}}$; therefore in the latter circuit it is constantly greater than in the former, and, in fact, a gradual transition takes place from equality of action, which is evinced when A disappears, to where the voltaic combination exceeds n times the action of the simple circuit, which case occurs when A is incomparably greater than n L. If by E we represent the relative length of the body upon which the circuit is to act by the force of its current, then from the above observations it results that it is most advantageous to employ a powerful simple circuit when A is very small in comparison to L; and on the contrary the voltaic battery, when A is very great in comparison with L.

But how in each separate case must a given voltaic apparatus be arranged so as to produce the greatest effect? Let us suppose, in solving this problem, that we possess a certain magnitude of surface,—for instance of copper and zinc,—from which at pleasure we can form a single large pair of plates, or any number of smaller pairs, but in the same proportion, and, moreover, that the liquid between the two metals is constantly the same, and of the same length, which latter supposition means nothing more than that the two metals between which the liquid is confined retain, under all circumstances, the same distance from each other.

Let $\varLambda$ be the reduced length of the body upon which the electric current is to act, L the reduced length of the apparatus when formed into a simple circuit, and A its tension; then when it is altered into a voltaic combination of x elements, its present tension will be x A, and the reduced length of each of its present elements x L, accordingly the reduced length of all the x elements x^2 L, consequently the magnitude of the action of the voltaic combination of x elements is

$$\frac{x\,\mathrm{A}}{x^2\,\mathrm{L} + \varLambda}.$$

This expression acquires its greatest value

$$\frac{\mathrm{A}}{2\sqrt{\varLambda\,.\,\mathrm{L}}} \text{ when } x = \sqrt{\frac{\varLambda}{\mathrm{L}}}.$$

We hence see that the apparatus in form of a simple circuit is most advantageous, so long as $\varLambda$ is not greater than L; on the contrary, the voltaic combination is most useful when $\varLambda$ is greater than L, and indeed it is best constructed of two elements when $\varLambda$ is four times greater than L, of three elements when $\varLambda$ is nine times greater than L, and so forth.

27. The circumstance that the current is of the same strength at all parts of the circuit, provides us with the means of multiplying its external action, as in the case where it is caused to influence the magnetic needle.

We will, for perspicuity, suppose that in order to test the action of the current on the magnetic needle, each time a part of the circuit be formed into a circle of a given radius, and so placed into the magnetic meridian that its centre coincides with the point of rotation of the needle.

Several such distinct coils, formed of the circuit in exactly the same manner, will, taken singly, produce, on account of the equality of the current in each, equally powerful effects on the magnetic needle; if we imagine them, therefore, so arranged near one another, that though they are separated by a non-conducting layer, they are yet situated so close together that the position of each one toward the magnetic needle may be regarded as the same, they would produce a greater effect on the needle in proportion as their number increased. Such an arrangement is called a *multiplier*.

Now, let A be the sum of the tensions of any circuit, and L its reduced length; let also Λ be the reduced length of one of the interposed conductors formed into a multiplier of n convolutions; then, if we represent the reduced length of one such convolution by λ, $\Lambda = n\,\lambda$, the action of the multiplier on the magnet needle will be proportional to the value $\frac{n\,A}{L + n\,\lambda}$.

But the action of a similar coil of the circuit, without the multiplier, is, according to the same standard $\frac{A}{L}$, and we will suppose the portion of the circuit, whence the coil is taken, to be of the same nature as in the multiplier; accordingly the difference between the former and the present effect is

$$\frac{n\,L - (L + n\,\lambda)}{L + n\,\lambda} \cdot \frac{A}{L},$$

which is positive or negative according as n L is greater or less than L $+ n\,\lambda$.

Consequently the action on the magnetic needle will be augmented or diminished by the multiplier formed of n coils, according as the n times reduced length of the circuit without the interposed conductor, is greater or less than the entire reduced length of the circuit with the interposed conductor.

If $n\,\lambda$ is incomparably greater than L, the action of the multiplier on the needle will be $\frac{A}{\lambda}$.

To this value, which indicates the extreme limit of the action by means of the multiplier (the galvanometer), whether it be strengthening or weakening, belong several remarkable properties, which we will briefly notice. It is understood that the multiplier is formed of so many coils that the magnitude of its action may, without sensible error, be considered equal to the limit value.

Since the action of a coil in the circuit is $\frac{A}{L}$, while the action of the multiplier, in connection with the same circuit, is $\frac{A}{\lambda}$, it is evident that the two actions are in the same ratio to each other as the reduced lengths λ and L; if, therefore, we are acquainted with the two actions, and with one of the two reduced lengths, the other may be found, and in the same manner one of the two actions may be deduced from the other, and the two reduced lengths.

Since the limit of the action of the multiplier is $\frac{A}{\lambda}$, it increases when λ is invariable in the same proportion as the sum of the tensions A in the circuit decreases; we may, therefore, by comparing the extreme actions of the same multiplier in various circuits, arrive at the determination of their relative tensions.

At the same time we perceive that the extreme action of the multiplier increases when several simple circuits are formed into a voltaic combination, and, indeed, in direct proportion to the number of the elements.

In this manner it is always in our power, in cases where the multiplier in connection with the simple circuit produces a weakening effect, to cause it to indicate any increase of force whatever.

If we call the actual length of a coil of the multiplier l, its conductibility χ, and its section ω, then $\lambda = \dfrac{l}{\chi\,\omega}$, and consequently the extreme action of the multiplier

$$\chi\,\omega\,.\frac{A}{l},$$

whence it results that in the same circuit the extreme actions of two multipliers of coils of equal diameter are in the ratio to each other of the products of their conductivities and their sections.

These extreme actions are, therefore, in two multipliers, which differ only in being formed of two distinct metals, in proportion to the conductivity of these metals; and when the multipliers consist of similar convolutions, and of one metal, their extreme actions are proportional to their sections.

But all these determinations are based upon the supposition that the action of a portion of the circuit on the magnetic needle, under otherwise similar circumstances, is proportional to the magnitude of the current.

Direct experiments have long since established the correctness of this supposition.

28. We will now consider a multiple conduction existing at the same time.

If, for instance, we imagine an open circuit, whose separated extremities are connected by several conductors, arranged side by side, it may be asked, by what law is the current distributed in the adjacent conductors? We might in answering this question proceed directly from the considerations contained in Sect. 11 to 13; but we shall more simply attain the required object from the peculiarity of voltaic circuits ascertained in Sect. 25, in which case to simplify, we will suppose that none of the former tensions are destroyed by the opening of the circuit, nor a new tension produced by the conductor which is introduced.

For if λ, λ', λ'', etc., represent the reduced lengths of the conductors brought into connection with the extremities of the open circuit, and α the difference of the electroscopic forces at the extremities of the circuit, after the conductors have been introduced, the same difference will also occur at the ends of the single adjacent conductors, since, according to the supposition we have made, no new tension is introduced by the conductor.

Since now according to Sect. 13, the magnitude of the current in the circuit must be equal to the sum of all the currents in the parallel conductors, we may regard the circuit as being divided into as many parts as there are parallel conductors; then, according to Sect. 25, the strength of the current in each adjacent conductor and in the corresponding part of the circuit, will respectively be

$$\frac{\alpha}{\lambda}, \quad \frac{\alpha}{\lambda'}, \quad \frac{\alpha}{\lambda''}, \text{ \&c.,}$$

whence, in the first place, it results that the strength of current in each branch conductor is in inverse ratio to its reduced length.

If we now imagine a single conductor of such character that, being substituted for all the parallel conductors in the circuit, the current therein remains unaltered; then in the first place, α, according to Sect. 25, must retain the same value, and if we designate by $\varLambda$ the reduced length of this conductor, must moreover be

$$\frac{1}{\varLambda} = \frac{1}{\lambda} + \frac{1}{\lambda'} + \frac{1}{\lambda''} + \text{\&c.}$$

From the preceding explanations we may conclude, that when A denotes the sum of all the tensions, and L the entire reduced length of the circuit without adjacent or parallel conductors, the strength of the current, while the adjacent conductors are in connection with the circuit, will be expressed in the circuit itself by $\frac{A}{L + \varLambda}$; in the joint conductor whose reduced length is λ, by $\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda}$; in the joint conductor whose reduced length is λ', by $\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda'}$; in the joint conductor whose reduced length is λ'', by $\frac{A}{L + \varLambda} \cdot \frac{\varLambda}{\lambda''}$; and so on, where for $\varLambda$ its value obtained from the equation

$$\frac{1}{\varLambda} = \frac{1}{\lambda} + \frac{1}{\lambda'} + \frac{1}{\lambda''} + \&c.,$$

has to be placed.

29. That in the above, the galvanic current, is found to be of equal strength at all points of the circuit, arises from the value of $\frac{du}{dx}$, deduced from the equation

$$u = \frac{A}{L} y - O + c,$$

being constant. This circumstance no longer happens if we start from the equations given in Sects. 22 and 23. In all these cases $\frac{du}{dx}$ is dependent on x, which indicates that the strength of the current is different at different places of the circuit.

We may hence draw the conclusion that the electric current is only of equal strength at all points of the circuit when it has assumed a permanent state, and when the circuit is not acted upon by the atmosphere.

This property likewise appears best adapted to enable us to find out, by experiment, whether the atmosphere exercises a perceptible influence on a voltaic circuit, or not; and we will therefore discuss this case at greater length.

Since according to Sect. 12, the strength of the electric current is given by the equation

$$S = \chi \omega . \frac{du}{dx},$$

we have only in each separate case to obtain the value of $\frac{du}{dx}$ from the equation found for the determination of the electroscopic force, and to place it in the one above.

Thus for a circuit which has assumed its permanent state, but upon which the surrounding atmosphere exercises no sensible influence according to Sect. 22,

$$u = \tfrac{1}{2}\,a\,.\,\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} - e^{-\beta l}} + \tfrac{1}{2}\,b\,\frac{e^{\beta x} + e^{-\beta x}}{e^{\beta l} + e^{-\beta l}},$$

where a represents the tension at the place of excitation, and h the sum of the electroscopic forces immediately adjacent on both sides of the place of excitation. We hence obtain

$$S = \chi\,\omega\,\beta\left(\tfrac{1}{2}\,a\,\frac{e^{\beta x} + e^{-\beta x}}{e^{\beta l} - e^{-\beta l}} + \tfrac{1}{2}\,b\,\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} + e^{-\beta l}}\right).$$

This expression gives the strength of the current at each place of the circuit; but the law, according to which the alteration of the current at various places of the circuit is effected, may be made more easily intelligible in the following manner:—

If, for instance, we differentiate the equation

$$S = \chi\,\omega\,\frac{d\,u}{d\,x},$$

we obtain the equation

$$\frac{d\,S}{d\,x} = \chi\,\omega\,\frac{d^2\,u}{d\,x^2};$$

and by multiplying both together,

$$\frac{d\,S}{d\,u} = \chi^2\,\omega^2\,\frac{d^2\,u}{d\,x^2}.$$

If now we substitute for $\frac{d^2\,u}{d\,x^2}$ its value $\beta^2\,u$, as obtained from the equation

$$0 = \frac{d^2\,u}{d\,x^2} - \beta^2\,u,$$

we have $\frac{d\,S}{d\,u} = \chi^2\,\omega^2\,\beta^2\,u$; and we hence obtain by integration

$$S^2 = c^2 + \chi^2\,\omega^2\,\beta^2\,u^2,$$

where c represents a constant remaining to be determined. If we designate by u'_1 the smallest absolute value which u occupies in the circumference of the circuit, and by

S′ the corresponding value of S, and determine in accordance with this, the constant c, we obtain

$$S^2 - S'^2 = \chi^2 \omega^2 \beta^2 (u^2 - u'^2).$$

It may be easily deduced from this equation that the current of a circuit which is influenced by the atmosphere is weakest where the electroscopic force without regard to the sign is smallest, and that it is of the same strength at places with equal but opposite electroscopic forces.

APPENDIX.

On the Chemical Power of the Galvanic Circuit. — 30. In the present Memoir we have constantly supposed that those bodies which are under the influence of the electric current remain unchangeable; we will now, however, take into consideration the action of the current on the bodies subjected to it, and the alterations in their chemical constitution thence resulting in any possible manner, as also the changes of the current itself produced by reaction.

To proceed on sure ground, let us return to what has been announced in Sects. 1 to 7, and connect our present considerations with those expressions and developments.

We will suppose two particles, and designate by s their mutual distance, by u and u' their electroscopic forces, which we admit to be of equal intensity in all points of the same particle; then, as may easily be deduced from what has been previously stated, the repulsive force between these two elements is proportional to the time $d\,t$, to the product $u'\,u'$, and to a function dependent on the position, size, and form of the two particles, which we will represent by F′; we accordingly obtain for the repulsive force between two particles the expression $F'\,u\,u'\,d\,t$.

If we here proceed in the same manner as in Sect. 6, and signify by the *moment of action* χ' between two places, the product of q' which expresses the force produced under perfectly determined circumstances between both, and its mean distance s', so that $\chi' = q'.s'$, and determine q' by putting $u = u' = 1$ in the expression $F'\,u\,u'\,d\,t$, and extending the action to the unit of time, we have $\chi' = F'\,s'$, whence it follows that $F' \frac{\chi'}{s'}$.

Let us now imagine, as we did in Sect. 11, the prismatic circuit to be divided into equally large, infinitely thin disks, and call M', M, M_1, those immediately following one another, which belong to the abscissæ $x + dx, x, x - dx$; then according to what has just been shown, the pressure which the disk M' exerts on the disk M is $F' u u' dt$; and if we admit that the position, size, and form of the particles remain in all disks the same, the counter pressure, which the disk M_1 exerts on the disk M, is $F' u u' dt$; the difference between these two expressions, viz.,

$$F' u (u' - u_1) dt,$$

gives accordingly the magnitude of the force with which the disk M tends to move along the axis of the circuit. This force acts contrary to the direction of the abscissæ when its value is positive, and in the direction of the abscissæ when it is negative.

If we substitute for $u' - u$ its value proceeding from the developments given in Sect. 11 for u' and u_1, the expression just found changes into the following:

$$2 F' u \frac{du}{dx} dx dt,$$

and if we take instead of the function F′ dependent on the nature of each single body, its value $\frac{\chi'}{s'}$, this expression, since s' is evidently here $d\,x$, changes into

$$2\,\chi'\,u\,\frac{d\,u}{d\,x}\,d\,t\,;$$

or if we reduce the moment of action χ', referring to the magnitude of the section ω, to the unit of surface, and at the same time extend the action to the unit of time, into

$$2\,\chi'\,\omega\,u\,\frac{d\,u}{d\,x},$$

where the present ′ represents the magnitude of the moment of action reduced to the unit of surface. If we write this latter expression thus:

$$2\,\frac{\chi'}{\chi}\,\chi\,\omega\,u\,\frac{d\,u}{d\,x},$$

in which χ denotes the absolute power of conduction of the circuit; and if we substitute for $\chi\,\omega\,\frac{d\,u}{d\,x}$, by which, according to the equation (*b*) in Sect. 12, the magnitude of the electric current is expressed, the sign S chosen for it, and i instead of $\frac{\chi'}{\chi}$, it is changed into $2\ i\ u$ S.

We hence perceive that the force with which the individual disks in the circuit tend to move, is proportional, both to their innate electroscopic force, and to the strength of the current, and that this force alters its direction at that place of the circuit where the electricity passes from the one into the opposite state.

And here occurs the circumstance which must not be overlooked, that this expression still holds, even when the electroscopic force u of the element M is changed in the moment of action, by any causes whatsoever, into any other abnormal U, while the electroscopic forces of the adjacent particles continue the same; only that in this case the value U must be substituted for u in the expression $2\ i\ u$ S.

It must also be observed that the expression $2\ i\ u$ S which we have found refers to the whole extent of the section

ω, which belongs to that part of the circuit which we have especially in view; if we wish to reduce this motive force of the circuit to the unit of surface, we must divide that expression by the magnitude of the section ω.

31. Without pursuing any further these conditions to an external change of place of the parts of a galvanic circuit, let us now turn to those changes in the qualitative state of the circuit which are produced by the electric current; i. e., in the internal relation of the parts to each other, and which derive their explanation from the electrochemical theory of bodies. According to this theory, compound bodies must be considered as a union of constituents which possess dissimilar electric states, or, in other words, dissimilar electroscopic force.

But this electroscopic force, quiescent in the constituents of the bodies, differs from that to which our attention has hitherto been directed, inasmuch as it is linked to the nature of the elements, and cannot pass from one to the other, without the entire mode of existence of the parts of the body being destroyed.

If we confine ourselves, therefore, in the following considerations, to the case where changes it is true occur in the quantitative relation of the constituents, and where consequently chemical changes of the body composed of these constituents also occur, but where the constituents themselves undergo no alteration without destroying their nature, we are able to show the validity of all the laws above developed of electric bodies with reference to their reciprocal attraction and repulsion, only the transition of the electricity from one particle to the other entirely disappears in the consideration of chemically different constituents.

A distinction here exists with reference to electricity exactly similar to that which we are accustomed to define relative to heat, by calling it sometimes latent, sometimes free heat.

For the sake of brevity, we will in like manner term that electroscopic force which belongs to the existence of the particles, which therefore they cannot part with, without at the same time ceasing to exist, the *electricity bound* to the bodies, or *latent electricity;* and *free electricity* that which is not requisite for the existence of the bodies in their individuality, and which therefore can pass from one element to the other, without the individual parts being on that account compelled to change their specific mode of existence for another.

32. From these suppositions advanced in electro-chemistry, in connection with what was stated in Sect. 30, respecting the mode in which voltaic circuits exert a different mechanical force on disks of different electrical nature, it immediately follows that when a disk belonging to the circuit is composed of constituents of dissimilar electric value, the neighboring disks will exert on these two constituents a dissimilar attractive or repulsive action, which will excite in them a tendency to separate, which when it is able to overcome their coherence, must produce an actual separation of constituents.

This power of the voltaic circuit, with

which it tends to decompose the particles into their constituents, we will call its *decomposing force,* and now proceed to determine more minutely the magnitude of this force.

Employing for this purpose all the signs introduced in Sect. 30, we will, moreover, imagine each disk to be composed of two constituents A and B, and designate by m and n the latent electroscopic forces of the constituents A and B, supposing the disk M to be occupied solely by one of the two, entirely excluding the other, in the same manner as u represents the free electroscopic force present in the same disk, and equally diffused over both constituents.

If we now admit that the two constituents A and B, before and after their union, constantly occupy the same space, and designate the latent electroscopic force corresponding to each chemical equivalent, contained in the disk M, and proceeding from the constituent A, by $m\ z$, then $n\ (1 - z)$ expresses the latent electroscopic force present in the same disk M, but originating from the constituent B: for the intensity of the force diffused over a body decreases in the same proportion as the space which the body occupies becomes greater, because by the increased distance of the particles from each other the sum of their actions, restricted to a definite extent, is diminished in the same proportion. But when two constituents combine, by both reciprocally penetrating one another, each extends beyond the entire space of the compound, on which account the intensity of the force proper to each constituent decreases by combination, in the same proportion as the space of the compound is greater than the space which each constituent occupied before the combination.

Consequently, if z denote the relation of the space which the constituent A, in the disk M, occupies previous to combination, to that space which the compound in the disk M occupied; and also, since we admit that both constituents, before and after the combination, occupy the same extent of space, $1 - z$ will denote the same relation relatively to the constituent B; then, since m and n designate the electroscopic forces of the constituents A and B previous to combination, $m\,z$ and $n\,(1-z)$ will represent the latent electroscopic force of the constituents A and B, which correspond to each chemical equivalent of the disk M; and, at the same time, it follows from the above, that the variable values z and $1-z$ cannot exceed the limits O and 1.

In order to ascertain the portion of the free electricity u pertaining to each constituent, we will assume that it is distributed over the single constituents in proportion to their masses. If, therefore, we represent respectively by α and β the masses of the constituents A and B, on the supposition that one alone, to the exclusion of the other, occupies the entire disk, then $\alpha\,z$ and $\beta\,(1-z)$ will represent the masses of the constituents A and B united in the disk M; consequently the portions

$$\frac{\alpha\,u\,z}{\alpha\,z+\beta\,(1-z)}, \quad \text{and} \quad \frac{\beta\,u\,(1-z)}{\alpha\,z+\beta\,(1-z)}$$

of the free electricity u appertain to the constituents A and B; instead of which, for the sake of conciseness, we will write

$$\alpha \, U \, z \text{ and } \beta \, U \, (1 - z).$$

If now we take into consideration what was stated in Sect. 30, respecting the motive force of the galvanic circuit, it is immediately evident that the tendency of the constituent A to move along the circuit is expressed by $2\,i\,(m + \alpha\,U)\,z\,S$, or that of the constituent B by $2\,i\,(n + \beta\,U)\,(1 - z)\,S$.

In both cases a positive value of the expression shows that the pressure takes place in an opposite direction to that of the abscissæ; a negative value, on the contrary, indicates that the pressure is exerted in the direction of the abscissæ.

To deduce from these individual tendencies of the constituents the force with which both endeavor to separate from each other, we must remember that this force is given by the twofold difference between the quantities of motion which each constituent would of itself assume were it combined with the other by no coherence, and those quantities of motion which each must assume were it strongly combined with the other. We thus readily find for the decomposing force of the circuit the following expression:

$$4\,i\,.\,z\,(1-z)\cdot\frac{m\,\beta-n\,\alpha}{\alpha\,z+\beta\,(1-z)}\cdot S,$$

from which we learn that the decomposing force of the circuit is proportional to the electric current, and also to a co-efficient dependent on the chemical nature of each place of the circuit.

If this expression has a positive value, it indicates that the separation of the constituent A takes place in a contrary direction to that of the abscissæ, that of the constituent B in the direction of the abscissæ; but if this expression has a negative value, it denotes a separation in the reverse direction. It is evident that the decomposing force of the circuit is constantly determined by the absolute value of the expression.

If $\alpha=\beta$, the decomposing force of the circuit changes into $4\,i\,.\,z\,(1-z)\,(m-n)\,.\,S$.

If $m z + n (1 - z) = 0$, viz., if the latent electroscopic forces existing in the united constituents are equal and opposed; or, what is the same, if the body, situated in the disk M, is perfectly neutral, in which case m and n have constantly opposite values, we obtain, for the decomposing force of the circuit, the following expression:

$$4\, i \,.\, \frac{m\, n}{m - n} \,.\, S\,.$$

The form of the general expression found for the decomposing force of the circuit shows that this force disappears; first, when $S = 0$; that is, when no electric current exists; secondly, when $z = 0$, or $z = 1$, i.e., when the body to be decomposed is not compound; thirdly, when $m \beta - n z = 0$; viz., when the densities of the constituents are proportional to the latent electroscopic forces which they possess, which circumstance can never occur with constituents of opposite electric nature.

All the expressions here given for the

decomposing force of the circuit refer to the entire section belonging to the respective place; if we wish to reduce the value of the decomposing force to the unity of surface, the expression must be divided by the magnitude of the section, to which attention has been already called in Sect. 30, in a similar example.

33. If this decomposing force of the circuit is able to overcome the cohesion of the particles in the disk, a cohesion produced by their electric opposition, this necessarily occasions a change in the chemical equivalent of the particles.

But such a change in the physical constitution of the circuit must at the same time react on the electric current itself, and give rise to alterations in it, with which a more accurate acquaintance is desirable.

To obtain this we will imagine a portion of the voltaic circuit to be a homogeneous fluid body, in which such a decomposition actually takes place; then, at all points of this portion, the elements of one kind will tend to move with greater force towards one side of the circuit than those of the other kind; and since we suppose that by the active forces the coherence is overcome, it follows, if we pay due attention to the nature of fluid bodies, that the one constituent must pass to one side, while the other on the contrary passes to the other side of the portion, which necessarily produces on one side a preponderance of the one constituent, and on the other side a preponderance of the other constituent.

But as soon as a constituent is predominant on one side of any disk, it will oppose by its predominance the movement of the like constituent in the disk towards the same side, in consequence of the repulsive force existing between both; the decomposing force, therefore, has now not merely to overcome the coherence between the two constituents in the disk, but also the reacting force in the neighboring disks. Two cases may now occur: the decomposing force of the electric current either instantly overcomes all the forces opposed to it, when the action terminates by a total separation of the constituents, the entire mass of the one passing to the one end of the portion, while the entire mass of the other constituent is impelled towards the other end of this portion; or such a relation takes place between the forces in action that the forces opposing the separation ultimately maintain the decomposing force in equilibrium; from this moment no further decomposition will occur, and the portion will be in a remarkable state, a peculiar distribution of the two constituents occurring, into the nature of which we will now inquire.

If we designate by Z the decomposing force of the current in any disk of the portion in the act of decomposition, Y the magnitude of the reaction by which the neighboring disks oppose the decomposition by the electric current, and X the force of the coherence of the two constituents in the same disk, then evidently the state of a permanent distribution within the supposed portion will be determined by the equation $X + Y = Z$; and it is already known from the preceding paragraph, that

$$Z = 4\, i\, z\, (1 - z)\ \frac{m\,\beta - n\,\alpha}{\alpha\, z + \beta\,(1 - z)} \cdot S;$$

or if we substitute $\chi\,\omega\,\frac{d\,u}{d\,x}$ for S,

$$z = 4\,\chi\,\omega\,\frac{d\,u}{d\,x} \cdot i\, z\,(1 - z)$$

$$\frac{m\,\beta - n\,\alpha}{\alpha\, z + \beta\,(1 - z)} \cdot$$

At the limits of the portion in question, we imagine the portion so constituted that insuperable difficulties there oppose themselves to any further motion; for it is obvious that otherwise the two extreme strata of both constituents, which it is clear could never of themselves reach equilibrium, would quit the portion in which we have hitherto supposed them, and either pass on to the adjacent parts of the circuit, or from any other causes separate entirely from the circuit.

We will not here follow the last-mentioned modification of the phenomenon any further, although it frequently occurs in nature, as sufficiently shown by the decomposition of water, the oxidation of the metals on the one side and a chemical change of a contrary kind occurring on the metals at the other side of the portion hitherto less observed, but placed entirely beyond doubt by Pohl's remarkable experiments on the reaction of metals. Besides, we will direct our attention to a difference which exists between the distribution of electricity above examined, and the molecular movement now under consideration. If, for instance, the same forces which previously effected the conduction of the electricity, and there, as it were, incorporeally without impediment strove against each other, here enter into conflict with masses by which their free activity is restricted, a restriction which whether we regard the electricity *de se ipso* as something material or not, must render their present velocities, beyond comparison, smaller than the former ones; therefore we cannot in the least expect that the permanent state which we at present examine will instantaneously occur like that above noticed, arising from the electric distribution; we have rather to expect that the permanent state resulting from the chemical equivalent of both constituents will make its appearance only after a perceptible, although longer or shorter time. We will now proceed to the determination of the separate valves X and Y.

34. To obtain the value X, we have merely to bear in mind that the intensity of coherence is determined by the force with which the two adjacent constituents attract or repel each other by virtue of their electric antagonism, and consequently, as was shown in Sect. 30, proportional to the product of the latent electroscopic forces $m\,z$ and $n\,(1 - z)$ possessed by the constituents of the disk M, and is moreover dependent on a function to be deduced from the size, form, and distance, which we will term $4\,\varphi$. Accordingly, when we restrict the coherence to the magnitude of the section ω,

$$X = -4\,\varphi\,m\,n\,z\,(1-z)\,\omega.$$

We have placed the sign — before the expression ascertained for the strength of the coherence, since a reciprocal attraction of the constituents only occurs when m and n have opposite signs; when they have the same signs the constituents exert a repulsive action on each other, which no longer prevents, but promotes, the decomposing force.

After this remark it becomes evident that a positive or negative value must be ascribed to the function φ, according as the expression taken for the decomposing force z is positive or negative; the sign of the function φ, therefore, changes when the direction of the decomposition is transposed from the one constituent to the other. The nature of the function φ is as little known to us as the size and form of the elements on which it is dependent; nevertheless, we may in our inquiries regard its absolute value as constant, since the size and form of the corporeal particles, acting on each other, must be conceived as being unchangeable as long as the two constituents remain the same, and the supposition that the two constituents constantly maintain for every chemical equivalent the same volume renders attention to the mutual distance of the chemically different particles unnecessary, as regard has already been paid, when determining the electroscopic forces in the disk M, to the relative distance of the elements of each constituent.

35. To determine the magnitude of the reaction Y, which in the disk M opposes the latent electricity of the neighboring disks to the decomposing force, we have nothing further to do than to substitute in the expression for z instead of u, the sum of all the latent electroscopic forces in the disk M.

Since now the sum of these latent forces is $m\,z + n\,(1-z)$, we obtain for the determination of the force Y, which is called into existence by the change in the chemical equivalent of the constituents, and which opposes the decomposition, after due determination of its sign, the following equation:

$$Y = 4\,\chi\,\omega\,\frac{d\,z}{d\,x}\,.\,i\,(n-m)\,.\,z\,(1-z)\,.\,\frac{m\,\beta - n\,\alpha}{\alpha\,z + \beta\,(1-z)}\,.$$

If now we substitute for χ, y and z, the values found in the equation

$$X + Y = Z,$$

we obtain, after omitting the common factor $4\ z\ (1 - z)$, and multiplying the equation by

$$\frac{\alpha\ z + \beta\ (1 - z)}{i\ (m\ \beta - n\ \alpha},$$

as the condition of the permanent state in the chemical equivalent of the two constituents, the equation

$$0 = \chi\ \omega\ \frac{d\ u}{d\ x} + \frac{\varphi\ m\ n}{i\ (m\ \beta - n\ x)} \times .$$
$$[\alpha\ z + \beta\ (1 - z)\ ]\ \omega -$$
$$\chi\ \omega\ (n - m)\ \frac{d\ z}{d\ x},$$

which, when we put

$$\frac{\varphi\ m\ n}{i\ (m\ \beta - n\ \alpha)} = \psi = \frac{\chi\ \varphi\ m\ n}{\chi'(m\ \beta - n\ \alpha)},$$

passes into

$$0 = \chi\ \omega\ \frac{d\ u}{d\ x} + \psi\ \ [\alpha\ z + \beta\ \ (1 - z)\ ] -$$
$$\chi\ \omega\ (n - m)\ \frac{d\ z}{d\ x}. \qquad (\text{ð})$$

This equation undergoes no change, as indeed is required by the nature of the subject, when m α, z and n, β, $1 - z$ are respectively interchanged, and at the same time the sign of φ is changed, as according to the remark made in the preceding paragraph, must take place since by this transformation the direction of the decomposition is transferred from one constituent to the other.

36. In order to be able to deduce from this equation the mode of the diffusion of the two constituents in the fluid, i.e., the value of z, we ought to know the power of conduction χ, and the electroscopic force u at each point of the portion in the act of decomposition, the values, however, of which are themselves dependent on that diffusion. Experience, as yet, leaves us in uncertainty respecting the change of conductivity, which occurs when two fluids are mixed in various proportions with one another, and likewise with respect to the law of tensions, which is followed by different mixtures of the same constituents in various proportion; for, if we do not

err, no experiments have been instituted relatively to the latter law, and the law of the change produced in the conducting power of a fluid, by the mixture of another, is not yet decidedly established by the experiments of Gay-Lussac and Davy. For this reason we have been inclined to supply this want of experience by hypothesis. We have, it is true, constantly endeavored to conceive the nature of the action in question, in its connection with those with whose properties we are better acquainted; but, nevertheless, we wish the determinations given to be regarded as nothing more than fictions, which are only to remain until we become by experiment in possession of the true law.

With regard to what relates to the change in the power of conduction of a body, by mixture with another, we have been guided by the following considerations.

We suppose two adjacent parts of a circuit of the same section ω, whose lengths are v and w, and whose powers of construction are a and b; then, when A is the sum of the tensions in the circuit, and L the reduced length of the remaining portion of the circuit, the magnitude of its current which results from the above found formulae, is

$$\frac{A}{L + \frac{v}{a\,\omega} + \frac{w}{\lambda\,\omega}}.$$

If now a conductor of the length $v + w$ and of the power of conduction χ with the same section, being taken instead of the two former, leaves the current of the circuit unchanged, then must

$$\frac{v}{a\,\omega} + \frac{\omega}{b\,\omega} = \frac{v + w}{\chi\,\omega},$$

whence we find

$$\chi = \frac{a\,b(v + \omega)}{b\,v + a\,w}.$$

But it is perfectly indifferent for the magnitude of the current, whether the entire length v be situated near the entire length c, or any number of the disks be formed of the two, which are arranged in any chosen order, if only the extreme parts remain of the same kind, as otherwise a change might result in the sum of the tensions, consequently also in the strength of the current. If we extend this law (which holds for any mechanical mixture) likewise to a chemical compound, the above value found for χ evidently gives the conducting power of the compound, where, however, it has been taken for granted that the two parts of the circuit, even after the mixture, still occupy the same volume, for v and w are here evidently proportional to the spaces occupied by the two bodies mixed with each other.

If we now apply this result to our subject, and therefore put, instead of v and w, the values z and $1 - z$, which express the relations of space of the two constituents of the disk M, we obtain, when a denotes the conducting power of the one constituent A, and b the same for the constituent B; further, χ the power of conduction of the mixture of the two contained in the disk M, the following expression for χ:

$$\chi = \frac{a\,b}{a + (b - a)\,z}.$$

37. Having thus determined the power of conduction at each place of the extent in the act of decomposition, there only remains to be ascertained the nature of the function u at each such place; and since all tensions and reduced lengths in the part of the circuit in which no chemical change occurs are unalterable and given, it is, in accordance with the general equation given in Sect. 18, which likewise holds for our present case, only requisite for the perfect knowledge of the function u, that we are able to determine the tensions and reduced lengths for each place within the extent in which the chemical change takes place.

But evidently the reduced length of the disk M is $\frac{d\,x}{\chi\,\omega}$;

or if we substitute for χ its value just found,

$$\frac{a + (b - a)\,z}{a\,b\,\omega}\,d\,x;$$

we accordingly obtain the reduced length of any part of that extent, if we integrate the above expression, and take the limits of the integral corresponding to the commencement and end of the part. If now we bear in mind that the integral

$$\int \frac{a + (b - a) z}{a\, b\, \omega} d\, x$$

may also be written thus:

$$\frac{l}{b\, \omega} + \frac{b - a}{a\, b\, \omega^2} \int h\, \omega\, d\, x,$$

when l represents the length of the part over which the integral is to be extended, and $z\, \omega\, d\, x$ expresses merely the space which the constituent A in the disk M occupies; consequently $\int h\, \omega\, d\, x$, the sum of all the spaces which the constituent A fills in the part whose reduced length has to be found, it is obvious that the reduced length of the entire portion in the act of decomposition remains unchangeable during the chemical change, since, as we have supposed, each constituent maintains, under all circumstances, constantly the same volume. The same result may also by directly deduced from what was advanced in the preceding paragraph; however, this unchangeability only relates to the reduced length of the entire portion; the reduced length of a *part* of it does not in general depend merely on the actual length of this part, but likewise on the contemporaneous chemical distribution of the constituents in the extent, and must, therefore, in each separate case, be first ascertained in the manner indicated.

38. We have lastly to determine the alteration in the tension of the circuit, which is produced by the chemical alteration of the extent, which has hitherto been considered. For this purpose we assume, till experience shall have taught us better, the position, that the magnitude of the electric tension between two bodies is proportional, first to the difference of their latent electroscopic forces, and secondly to a function, which we will term the *co-efficient of the tension*, dependent on the size, position, and form of the particles which act on each other at the place of contact. Not only from this hypothesis may be deduced the law which the tensions

of the metals observe *inter se*, — nothing further being requisite than to assume the same coefficient of tension between all metals placed under similar circumstances, — but it likewise affords an explanation of the phenomenon, in accordance with which the electric tension does not merely depend on the chemical antagonism of the two bodies, but also on their relative density, and can for this reason exhibit themselves differently, even in different temperatures.

For the same reasons which we have already mentioned in Sect. 34 on the determination of the coherence which occurs between the two constituents of a mixed body, we shall likewise admit here, in the circumference of the chemically variable extent as constant, the unknown function dependent on the size, form, and position of the particles in contact and designate it by φ'. Since now the latent electroscopic force in the disk M, to which the abscissa v belongs, is expressed by

$$n + (m - n) z,$$

and that in the disk M′, to which the abscissa $x + dx$ belongs, by

$$n + (m - n)\,z + (m - n)\,dz,$$

the tension originating between the disks M and M′ is

$$-\varphi'\,(m - n)\,dz;$$

consequently the sum of all the tensions produced throughout a portion exposed to chemical change

$$-\varphi'\,(m - n)\,(z''\,z'),$$

when z' and z'' represent those values of z which belong to the commencement and end of the extent in question.

But the tension of the circuit undergoes, besides the change just explained, a second one, from the extremities of the chemically changeable portion, which are in connection with the other chemically unchangeable parts of the circuit, undergoing a gradual change during the decomposition till they arrive at their permanent state, giving rise at those places to an altered tension.

If, for instance, we call ζ the value of

z which belong to all places of the extent in question, before chemical change has begun in it, and designate the co-efficient of the tension occurring at the extremities of this extent, supposing that it is the same at both ends, by φ'', and moreover express by μ and ν the latent electroscopic forces of those places of the chemically unalterable part of the circuit which are situated adjacent to the chemically changeable extent, the tensions existing at these places can be determined individually. They are, namely, previous to the commencement of chemical change, the following:

$$\varphi'' [\mu - (n + (m - n) \zeta)],$$
and $$\varphi'' [(n + (m - n) \zeta) - \nu];$$

and after the permanent state in the decomposition has been attained, if we, as above, let z' and z'' be those values of z which belong in this state to those places, they are the following:

$$\varphi'' [u - (n + (m - n) z')],$$
and $$\varphi'' [(n + (m - n) z'') - \nu],$$

their sum is therefore in one case

$$\varphi'' (\mu - \nu),$$

and in the other

$$\varphi'' (\mu - \nu) + \varphi''(m - n(z'' - z');$$

consequently the increase of tension at those places is

$$\varphi'' (m - n) (z'' - z').$$

If we add this change of the tension to that above found, we obtain for the entire difference of the tension, produced by the decomposition until the commencement of the permanent state,

$$(\varphi'' - \varphi') (m - n) (z'' - z'),$$

which, if we substitute φ for $\varphi'' - \varphi$, changes into

$$\Phi(n - m) (z'' - z').$$

If now we represent by S the strength of the current, and by A the sum of the tensions in the circuit before any chemical change has commenced, by S′ the strength of the current after the permanent state has been attained; lastly, by L the reduced length of the entire circuit, which, as we have seen, remains under all circumstances the same, it results

$$S' = \frac{A - \Phi\,(n-m)\,(z''-z'}{L},$$

or if we write for $\frac{A}{L}$ its equivalent S,

$$S' = S - \frac{\Phi\,(n-m)\,(z''-z')}{L},$$

so that therefore, $\frac{\Phi\,(n-m)\,(z''-z)}{L}$

designates the decrease produced in the strength of the current by the chemical alteration.

39. We now proceed to the final determination of the chemical alteration in the changeable portion, and the change of the current in the whole circuit produced thereby, where, however, we have constantly to keep in view only the permanent state of the altered portion. If we substitute in the equation (δ) given in Sect. 35, for $\chi\,\omega\,\frac{du}{dx}$ its value S′, which, as we have just found, is solely dependent on the fixed and unalterable values of z, and there-

fore has to be treated in the calculation as a constant magnitude; further, for χ its value $\frac{a\,b}{a + (b - a)\,z}$, given in Sect. 36, this equation changes into

$$0 = S' + \Psi\,\omega\,\beta + \Psi\,\omega\,(\alpha - \beta)\,z - \frac{a\,b\,\omega\,(n - m)}{a + (b - a)\,z} \cdot \frac{d\,h}{d\,x};$$

or if we place $S' + \Psi\,\omega\,\beta = \Sigma$, and $\Psi\,\omega\,(\alpha - \beta) = \Omega$, into

$$0 = \Sigma + \Omega\,z - \frac{a\,b\,\omega\,(n - m)}{a + (b - a)\,z} \cdot \frac{d\,z}{d\,x},$$

from which by integration we deduce the following:

$$c = \frac{(b - a)\,\Sigma - a\,\Omega}{a\,b\,\omega\,(n - m)}\,x + \log \frac{\Sigma + \Omega\,z}{a + (b - a}\,z,$$

where c represents a constant remaining to be determined. If we designate by X the abscissa of that place of the chemically changed portion for which z has still the same value, which previous to the commencement of the chemical decomposition belonged to each place of this portion, for which therefore $z = \zeta$, and determine in accordance with this statement the constant c, our last equation acquires the following form:

$$\frac{\Sigma + \Omega z}{a + (b - a) z} = \frac{\Sigma + \Omega \zeta}{a + (b - a) \zeta} . e^{\frac{(b-a)\Sigma - a\,\Omega}{a\,b\,\omega\,(n-m)}(X - x,)}$$

where e denotes the base of the natural logarithms.

The following consideration leads to the determination of the value X.

Since, namely, ζ represents the space which the constant A occupies in each individual disk of the changeable portion previous to the commencement of the chemical decomposition, if we denote by l the actual length of this portion, $l\,\zeta$ expresses the sum of all the spaces which the constituent A occupies on the entire expanse of the changeable portion; but this sum must constantly remain the same, since, according to our supposition, no part of either of the constituents is removed from this portion, and both maintain, under all circumstances, the same volume, even after chemical decomposition has taken place; we obtain therefore

$$l\,\zeta = \int z\,d\,x,$$

wherefore z is to be substituted its value resulting from the previous equation, and the abscissæ corresponding to the commencement and end of the changeable portion are to be taken as limits of the integral. These two last equations, in combination with that found at the end of the previous paragraph, answer all questions that can be brought forward respecting the permanent state of the chemical alteration, and the change in the electric current thus produced, and so form the complete base to a theory of these phenomena, the completion of the structure merely awaiting a new supply of materials from experiment.

40. At the conclusion of these investigations we will bring prominently forward a particular case, which leads to expressions that on account of their simplicity, allow us to see more conveniently the nature of the changes of the current produced by

the chemical alteration of the circuit. If, for instance, we admit $a - b$, and $\alpha - \beta$, the differential equation obtained in the preceding paragraph changes into the following:

$$O \Sigma \, d\, x - a\, \omega\, (n - m)\, d\, z,$$

whence we obtain by integration

$$z - \zeta = \frac{\Sigma\, (x - \psi)}{a\, \omega\, (n - m},$$

when X designates the value of x for which $z = \zeta$. Since in this case the value of z constantly changes to the same amount on like differences of the abscissæ, the abscissa X, which belongs to its mean value ζ, as it was at all places of the changeable portion previous to the commencement of the chemical decomposition, must be referred to the middle of this portion. If, therefore, z' and z'' as above represent the values of z which correspond to the beginning and end of the variable portion, and l the actual length of this portion, it follows, from our last equation, that

$$z'' - \zeta = + \tfrac{1}{2} \, \frac{l\, \Sigma}{a\, \omega\, (n - m)},$$

and

$$z' - \zeta = -\tfrac{1}{2}\ \frac{l\,\Sigma}{a\,\omega\,(n-m)},$$

and from these two equations result

$$(n-m)\,(z''-z') = \frac{l}{a}\,.\,\Sigma;$$

or, if we put instead of $\frac{l}{a\,\omega}$ by which here nothing further is expressed than the unchangeable reduced length of the chemically variable portion, the letter λ, the following:

$$(n-m)\,(z''-z') = \lambda\,\Sigma.$$

If we place this value of $(n-m)$ $(z''-z')$ in the equation found in Sect. 38,

$$S' = S - \frac{\Phi\,(n-m)\,(z''-z')}{L}$$

and at the same time substitute for Σ its value $S' + \psi\,\omega\,\alpha$, we obtain

$$S' = S - \frac{\Phi\,\lambda}{L}\,(S' + \psi\,\omega\,\alpha),$$

an equation, the form of which is extremly well suited to indicate in general the nature of the change of the current produced by the chemical alteration, and the expressions of which coincide exceedingly well with the numerous experiments I have made on the fluctuation of the force in the hydro-circuit, and of which a small part only has been published.[1]

The principal points thus beautifully explained by Ohm's theoretical investigations are the following: —

(1) That the current of electricity traversing a given circuit is directly proportional to the difference of potential at the two ends of the circuit.

(2) That the current is inversely proportional to the length of the wire, so that for instance the current from a given source circulating in two miles of wire will only be half as strong as when it circulates in but one mile of the same wire.

(3) That the current is directly proportional to the sectional area of the wire, or, in other words, to the square of its diameter, or to its weight per mile.

[1] Schweigger's Jahrbuch, 1825, Part 1, and 1826, Part 2.

(4) That the current is directly proportional to the specific conductivity of the material of whieh the wire is composed; or, as it is more frequently stated, it is inversely proportional to its specific resistance.

(5) The potential of a circuit connected to a source of electricity is highest at the positive pole of the source, and lowest at its negative pole. The potential falls uniformly through the whole circuit, when the conductor is of uniform size and material; and otherwise falls uniformly through uniform resistances.

Ohm's law then is simply that:

$$\text{Current} = \frac{\text{Electromotive Force}}{\text{Resistance}} \text{ or } C = \frac{E}{R.}$$

It may otherwise be stated as:

Electromotive Force = Current $\times$ Resistance, or $E = C \times R$.

And as

$$\text{Resistance} = \frac{\text{Electromotive Force}}{\text{Current}} \text{ or}$$

$$R = \frac{E}{C},$$

and the unit of Resistance has been named the Ohm in honor of the demonstrator of this law.

KIRCHOFF'S LAWS.

Kirchoff supplemented Ohm's law by two others; viz.—

(1) "In any network of conductors containing currents, the algebraical sum of the currents meeting at any one point is zero; the currents being reckoned positive or negative according as they flow towards or away from that point or *vice versa.*"

This may be symbolically written "$\Sigma c = 0$."

(2) "In a network of conductors containing currents and E. M. F.'s. if any closed circuit be taken, the algebraical sum of the products of the current in each conductor into the resistance of that conductor taken round that circuit is equal to the algebraical sum of the E. M. F.'s. acting round that circuit. Currents flowing or E. M. F.'s. acting round the circuit in one direction

being taken as positive, those in the reverse direction as negative."

And this law may be written

$$\Sigma\, c\, R = \Sigma\, E.$$

Bosscha's Corollaries.

Certain corollaries have been drawn by Bosscha from the above, with which we will conclude. These are: —

(1) If in any system of circuits containing any electromotive forces there is a conductor in which the current is zero, the currents in the remaining circuits are not altered, if the circuit of the conductor in question is taken away altogether with the electromotive force which is contained in it.

(2) If the conductor in question contains no electromotive force at all, then after it has been withdrawn we may connect the terminal points *m* and *n* directly with each other, without changing by this means the remaining currents. If, on the contrary, it contains an electromotive force, the

points can only be joined again, by inserting between them the equivalent electromotive force.

(3) If in a system of linear conductors there are two wires, *a* and *b*, in which an electromotive force in *a* produces no current at *b*, then the wire *a* may be divided without changing the intensity at *b*; and likewise without altering the intensity at *a*, the wire *b* may be divided.