District of Columbia, with the possible inclusion of Virginia. Professor Abraham Cohen, of Johns Hopkins University, is the secretary.

(4) A committee consisting of Professor Huntington, chairman, Professor Cajori and the secretary-treasurer was appointed with power to determine the time and place of the summer meeting, in conference with a similar committee of the American Mathematical Society.

(5) It was voted to appoint a committee which should in conjunction with a similar committee of the Society consider the question of possible assistance for *Revue Semestrielle* and the *Jahrbuch über die Fortschritte der Mathematik*. The committee was empowered to include also in its investigation other international projects of a kind similar to the two named. Mathematicians the country over are feeling increasingly the deplorable influence of the European war as it affects such indispensable aids as the German and French encyclopædias, the two journals above mentioned, and similar reference books. This action has been taken in order that the two great mathematical organizations of America may consider what contribution they may perhaps make in rendering assistance to these valuable journals of record.

(6) It was voted to hold the next annual meeting in Chicago in conjunction with the Chicago meeting of the American Mathematical Society.

(7) In a session following the election of officers, the Council, in pursuance of its constitutional authority to fill vacancies *ad interim*, filled the vacancy caused by the election of Professor Cajori to the presidency by the appointment of Professor E. V. Huntington, to serve until January, 1918.

(8) The members of the Committee on Publications (H. E. Slaught, managing editor, R. D. Carmichael, and W. H. Bussey) were reappointed for the year 1917.

(9) A Committee on Membership with ex-President Hedrick as chairman was authorized by the Council.

(10) The president-elect was empowered to make the necessary modifications in the existing committees of the Council and to appoint the new committees already authorized. He has accordingly appointed the following:

Committee on Sections: D. E. Smith, Chairman; E. R. Hedrick, M. B. Porter.

Committee on Membership: E. R. Hedrick, Chairman; E. V. Huntington, M. W. Haskell.

W. D. CAIRNS, *Secretary-Treasurer.*

ON THE ORIGIN OF CERTAIN TYPICAL PROBLEMS.[1]

By DAVID EUGENE SMITH.

One thing which impresses the student of mathematical problems is that several which he would naturally classify as purely fictitious and of the nature of pleasing puzzles apparently had their origin in genuine applications of mathematics to questions of real life. Of these I shall mention only four, although the list could be greatly extended.

[1] Extract from a paper on the History of Mathematical Recreations, read before the Mathematical Association of America at Cambridge, Mass., September 1, 1916.

The first of these problems, without which an algebra of to-day might by some be thought to be incomplete, so rooted is it in our traditions, is that of the pipes filling the cistern. No problem has had a longer and more continuous history, and the traveler who is familiar with the Mediterranean lands cannot fail to recognize that here is its probable origin. Not a town of any size that bears the stamp of the Roman power is without its public fountain into which or from which several conduits lead. In the domain of physics, therefore, this would naturally be the most real of all the problems that came within the purview of every man, woman, or child of that civilization. Furthermore, the elementary clepsydra[1] may also have suggested the same line of problems, the principle involved being the same.

The problem in definite form first appears in Heron's Μετρήσεις of about 100 A. D., and although there is some question as to the authorship and date of the work, there is none as to the fact that this style of problem would appeal to such a writer as he. It next appears in the writings of Diophantus, c. 275 A. D.,[2] and among the Greek epigrams attributed to Metrodorus, c. 325 A. D., and soon after this it became common property in the east as well as the west. It is found in the list attributed to Alcuin (c. 825); in the great classic of India, the *Lilāvati* of Bhāskara[3] (c. 1150); in the best-known of all the Arab works on arithmetic, the *Kholāsat-el-hisāb* of Behā-ed-dīn (1547–1622); and in numerous medieval manuscripts. When books began to be printed it was looked upon as one of the stock problems of the race, and many of the early writers gave it a prominent position, among them being men like Petzensteiner (1483), Tonstall (1522), Gemma Frisius (1540), and Robert Recorde (c. 1540).[4]

Such, then, was the origin of what was once a cleverly stated problem of daily life. There is, however, this interesting law of book writers—that most of them will steal from one another without the least scruple if they can thinly veil the theft. This problem, therefore, like dozens of others, went through many metamorphoses, of which I shall mention only a few.

In the fifteenth century, and very likely much earlier, there appeared the variant of a lion, a dog, and a wolf, or other animals, eating a sheep,[5] and this form was even more common in the sixteenth century.[6]

[1] Attributed to Plato but improved by Ctesibus of Alexandria. On the whole subject of clepsydræ see Marquardt, J., *La vie privée des Romains*, French edition, Paris, 1893, p. 458.

[2] In Bachet's edition (the Fermat edition of 1670, p. 271) appears this metrical translation:

Totum implere lacum tubulis è quatuor, uno
Est potis iste die, binis hic & tribus ille,
Quatuor at quartus.
Dic quo spatio simul omnes.

[3] See Taylor's translation, p. 50; Colebrooke translation, p. 42.

[4] In Recorde it appears for the first time in English: "Ther is a cestern with iiij. cocks, conteinyng 72 barrels of water, And if the greatest cocke be opened, the water will auoyde cleane in vj howers," etc. *Ground of Artes*, 1558 edition, folio A, 7 v.

[5] Johann Widman (1489) under the chapter title "Eyn fasz mit 3 zapffen." His form is:

"Lew Wolff Hunt Itm̄ des gleichen 1 lew vnd 1 hunt vn̄ 1 wolff diese essen mit einander 1 schaff. Vnd der lew esz das schaff allein in einer stund. Vnd d' wolf in 4 stunden. Vnd der hunt in 6 stunden. Nun ist die frag wan sy dass schaff all 3 mit einād̄er essen / in wie lāḡer zeit sy das essen." 1509 edition, folio 92; 1519 edition, folio 112.

[6] Thus Cataneo, *Le Pratiche*, 1546; Venice edition of 1567, folio 59v: "Se un Leone mangia

In the sixteenth century we also find in various books the variant of the case of men building a wall or a house, in place of pipes filling a cistern, and this form has survived to the present time. It appeared in Tonstall's exhaustive treatise, *De Arte Supputandi*, in 1522,[1] in Cataneo's well-known work of 1546,[2] and in due time became modified to the form beginning, "If A can do a piece of work in 4 days, B in 3 days," and so on.

The influence of the wine-drinking countries shows itself in the variant given by that remarkable writer Gemma Frisius (1540),[3] who states that a man can drink a cask of wine in 20 days, but if his wife drinks with him it will take only 14 days, from which it is required to find the time it would take his wife alone.

The influence of a rapidly growing commerce led one of the German writers of 1540 to consider the case of a ship with 3 sails, by the aid of the largest of which a voyage could be made in 2 weeks; with the next in size in 3 weeks, and with the smallest in 4 weeks, it being required to find the time if all three were used, several factors being evidently ignored, such as one sail blanketing the others and the speed not being proportional to the power.[4]

The agricultural interests changed it to a mill with four "Gewercken,"[5] and other interests continued to modify it further until, as is usually the case, the style of problem has tended to fall from its own absurdity. Merely mentioning one of our modern writers who modifies the problem to the case of the pipes of a gasolene tank in a motor car, I may close its varied history by referring to a writer of the early nineteenth century,[6] moved by a bigotry which we would not countenance in academic circles to-day, who proposed to substitute priests praying for souls in purgatory.

Thus we see a recreative problem, starting as an ingenuously worded practical case, becoming fictitious under changed conditions, maintaining itself for two thousand years because of its recreative feature, and almost falling by the wayside because of the absurdities which finally attached to it. It is likely to retain, however, some minor place in our schools because it is not only real within the imagination of pupils, which our technical mechanical problems usually are not, but it is interesting and illustrates a valuable mathematical principle.

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in 2. hore una pecora, & l'Orso la mangia in 3. hore, & il Leopardo la mangia in 4. hore, dimandasi cominciando a mangiare una pecora tutti e 3. a un tratto in quanto tempo la finirebbono."

This form is also found in J. Albert's work of 1540 (1561 edition, folio Nviii), in Coutereel (1631 edition, p. 352) and in the works of numerous other writers.

[1] With the statement that it is similar to the one about the cistern pipes: "Questio hæc similis est illi de cisterna tres habete fistulas: et simili modo soluenda." Folio f. 1.

[2] See folio 60v of the Venice edition of 1567.

[3] 1563 edition of his arithmetic, folio 38.

[4] "Item / 1 ein Schiff mit 3 Siegeln gehet vom Sund gen Riga / Mit dem grösten allein / in 2 wochen / Mit dem andern / in 3 wochen / Vnnd mit dem kleinsten / in 4 wochen," etc. J. Albert (1540), 1561 edition, folio Nvii.

[5] "Ein Mülmeister hat ein Müle mit vier Gewercken / Mit dem ersten mehlt er in 23 stūden 35 Scheffel / Mit dem andern 39 Scheffel / Mit dem dritten 46 Scheffel / Vnnd mit dem vierten 52 Scheffel," etc. The question then is how long it will take them together to grind 19 Wispel (1 Wispel = 24 Scheffel). *Ibid.*

[6] Hay, *The Beauties of Arithmetic*, 1816, p. 218.

The next problem to which I wish to call your attention has not maintained its place in our books although it has an honorable history of over 2,000 years; it is interesting, it is real within the realm of the pupil's imagination; but it fails for the reason that no principle is involved that is needed in secondary mathematics. The problem is the one commonly known as the Josephsspiel, or the one of the Turks and Christians. It relates that 15 Turks and 15 Christians were on a ship and that half had to be sacrificed; it being necessary to choose the victims by lot, the question is as to how they can be arranged in a circle so that, in counting round, every fifteenth should be a Turk.

It is probable that the problem goes back to the custom of *Decimatio* in the old Roman armies, the selection by lot of every tenth man when a company had been guilty of cowardice, mutiny, or loss of standards in action. Both Livy (ii, 59) and Dionysius (ix, 50) speak of it in the case of the mutinous army of the consul Appius Claudius (B. C. 471), and Dionysius further speaks of it as a general custom. Polybius (vi, 38) says that it was a usual punishment when troops had given way to panic. The custom seems to have died out for a time, for when Crassus resorted to decimation in the war of Spartacus he is described by Plutarch (Crassus, 10) as having revived an ancient punishment. It was extensively used in the civil wars and was retained under the Empire, sometimes as *vicesimatio* (every twentieth man being taken), and sometimes as *centesimatio* (every hundredth man).

Now it is very improbable that those in charge of the selection would fail to have certain favorites, and hence it is natural that there may have grown up a scheme of selection that would save the latter from death. Such customs may depart, but their influence remains in various ways. In the present great war we have frequently read of a regiment being decimated; but how few of us have thought of the origin of the expression.[1]

In its semi-mathematical form it is first referred to in the work of an unknown author, possibly Ambrose of Milan, who wrote, under the nom de plume of Hegesippus, a work *De bello iudaico*.[2] In this work he refers to the fact that Josephus, the author of the well-known history of the wars of the Jews, was saved on the occasion of a choice of this kind.[3] Indeed, Josephus himself refers to the matter of his being saved by lucky chance or by the act of God.[4]

The oldest European trace of the problem, aside from that of Hegesippus, is found in Codex Einsidelensis No. 326, of the beginning of the tenth century. It is also referred to in a manuscript of the eleventh century now in the Munich library and in Codex Bernensis No. 704, of the twelfth century. It is given in the *Ta'hbula* of Rabbi ben Esra (d. 1167) in the twelfth century, and indeed

[1] Lucas, in his *Arithmétique Amusante*, p. 17, also suggests the origin of the problem in the custom of *decimatio*.

[2] Edited by C. F. Weber and J. Caesar, Marburg, 1864. See Ahrens, Math. Unterh. u. Spiele, p. 286.

[3] "Itaque accidit ut interemtis reliquis Iosephus cum altero superesset neci." Quoted from Ahrens, l. c.

[4] Καταλείπεται δὲ οὗτος, εἴτε ὑπὸ τύχης χρὴ λέγειν εἴτε ὑπὸ Θεοῦ προνοίας σὺν ἑτερῳ.

it is to this writer that Elias Levita, who seems first to have given it in printed form (1518), attributes its authorship.

The problem, as it came to be stated, related that Josephus at the time of the sack of the city of Jotapata by Vespasian, hid himself with forty other Jews in a cellar. It becoming necessary to sacrifice some of the number, a method analogous to the old Roman method of *decimatio* was adopted, but in such way as to preserve himself and a special friend. It is on this account that the Germans still call the problem by the name of Josephsspiel.

Chuquet (1484) mentions the problem, as does at least one other writer of the fifteenth century.[1] When, however, printed works on algebra and higher arithmetic began to appear, it became well known. The fact that such writers as Cardan[2] and Ramus[3] gave it prominence was enough to assure its coming to the attention of scholars.[4]

Like so many curious problems, this one found its way to the Far East, appearing in the Japanese books as relating to a mother-in-law's selection of the children to be disinherited. With characteristic Japanese humor, however, the woman was described as making an error in her calculations so that her own children were disinherited and her step-children received the estate.[5]

The third problem of which I think the origin is worth our attention is the common one of the testament. It relates that a man about to die made a will bequeathing $\frac{1}{3}$ of his estate to his widow in case an expected child was a son, the son to have $\frac{2}{3}$; and $\frac{2}{3}$ to the widow if the child was a daughter, the daughter to have $\frac{1}{3}$. The issue was twins, one a boy and the other a girl, and the question was as to the division of the estate.

The problem in itself is of no particular interest, being legal rather than mathematical; but I mention it because it is a type and is by no means isolated. Under both the Roman and the Oriental influence these inheritance problems played a very important rôle in such parts of analysis as the ancients had developed. In the year 40 B. C. the *lex Falcidia* required at least $\frac{1}{4}$ of an estate to go to the legal heir. If more than $\frac{3}{4}$ was otherwise disposed of, this had to be reduced by the rules of partnership. Problems involving this "Falcidian fourth" were therefore common under the Roman law, just as problems involving the widow's dower right were and are common in the English law and in this country.

The problem as I have stated it appears in the writings of Juventius Celsus, a celebrated jurist of about 75 A. D., who wrote on testamentary law; in those of Salvianus Julianus, a jurist in the reigns of Hadrian (117–138), and Antoninus Pius (138–161), and in those of Cæcilius Africanus (c. 100), celebrated for his knotty legal puzzles.[6]

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[1] Anonymous MS. in Munich. See *Bibl. Math.*, 1893, p. 32; M. Curtze, *ibid.*, IX (2), 33; VIII (2), 116; X (2), 29; *Abhandlungen zur Geschichte der Math.*, III, 123.

[2] In his *Arithmetica* of 1539.

[3] In his edition of 1569, p. 125.

[4] It is also in Thierfelder's arithmetic (1587, p. 354), in Wynant van Westen's *Mathemat. Vermaecklyckh* (1644 edn. I, p. 16), in Wilken's arithmetic of 1669 (p. 395), and in many other early works.

[5] See Smith and Mikami, *History of Japanese Mathematics*.

[6] Coutereel (Eversdyck edition of 1658, p. 382) traces the problem back to lib. 28, title 2,

In the Middle Ages it was a favorite conundrum, and in the early printed arithmetics it is often found in a chapter on inheritances which reminds us of the Hindu mathematical collections.[1] It went through the same later development that characterizes most problems and finally fell on account of its very absurdity. That is, Widman (1489) takes the case of triplets, one boy and two girls,[2] and in this he is followed by Albert (1540) and Rudolff (1526).[3] Cardan (1539) complicates it by supposing 4 parts to go to the son and 1 part to the mother, or 1 part to the daughter and 2 parts to the mother, and in some way decides on an 8, 7, 1 division.[4] Texeda (1545) supposes 7 parts to go to the son and 5 to the mother, or 5 to the daughter and 6 to the mother,[5] while other writers of the sixteenth century complicate the problem even more.[6] The final complications of the "swanghere Huysvrouwe" or "donna grauida" are found in some of the Dutch books, and these and the change in ideas of propriety account for the banishment of the problem from books of our day.[7] The most sensible remark about the problem to be found in any of the early books is given in the words of the "Scholer" in Robert Recorde's *Ground of Artes* (c. 1540): "If some cunning lawyers had this matter in scanning, they would determine this testament to be quite voyde, and so the man to die vntestate, because the testament was made vnsufficient."[8]

The fourth problem to whose origin and development I wish to direct special attention is the one of pursuit. It would be difficult to conceive of a problem that would seem more real, since we commonly overtake a friend in walking, or are in turn overtaken. It would therefore seem very certain that this problem is among the ancient ones in what was once looked upon as higher analysis. We have a striking proof that this must be the case in the famous paradox of Achilles and the Tortoise, the history of which has been so carefully and entertainingly worked out by our colleague, Professor Cajori. It is a curious fact, however, that it is not to be found in the Greek collections, although it must also be said that we have not a single work on the Greek *logistice* (λογιστική) extant, so that

law 13 of the *Digest* of Julianus. He gives the usual 4, 2, 1 division as followed by Tartaglia, Rudolff, Forcadel, Ramus, Trenchant, vander Schuere, Mellema, and vander Gucht. Coutereel, however, argues for the 4, 3, 2 division, and in this he has the support of Anth. Smijters. Peletier gives 2, 2, 1, and Chauvet gives 9, 6, 4. Brief historical notes appear in other books, as in the Schonerus edition of Ramus (1586 edition, p. 186).

[1] Thus we have "Ein Testament" (Widman), "Erbteilung vnd vormundschaft" (Riese), "Erf-Deelinghe" (Vander Schuere), and "Erbtheilugs-Rechnung" (Starcken).

[2] Edition of 1558, folio 97. He then divides the property in the proportion 4, 2, 1, 1.

[3] Unger, p. 109.

[4] *Arithmetica*, cap. 66, ex. 87.

[5] Folio Xliij.

[6] Ghaligai (1552, folio 65), Köbel (1518, folio Fij), Riese (*Rechnung nach lenge*, 1550, folios 43, 100), Trenchant (1571, 1578 edition, p. 328), Vander Schuere (1600, folio 96), Peletier (1607 edition, p. 244), Coutereel (1631 edition, p. 358), Starcken (1714 edition, p. 444), Tartaglia (*Tvtte l'opere d'aritmetica*, 1592 edition, II, p. 136).

[7] "Soo ontfangt sy ter tijdt haerder baringhe eenen Sone met een Dochter / eñ een Hermaphroditus, dat is / half Man / half Vrouwe." Vander Schuere, 1600, folio 98. In this case he divides 3175 guldens thus: d. 254, m. 508, s. 1524, h. 889. The same problem appears in Clausberg, *Demonstrative Rechen-Kunst*, 1772.

[8] 1558 edition, folio X 8.

it may have been common without our knowing of the fact. It appears, however, among the *Propositiones ad acuendos juvenes* attributed to Alcuin, in the form of the hound pursuing the hare,[1] and thereafter it was looked upon as one of the stock questions of European mathematics. I have run across it in an Italian manuscript of c. 1440, it is in Petzensteiner's work of 1483,[2] Calandri used it in 1491,[3] Pacioli gives it in his Suma of 1494,[4] and most of the writers of any prominence in the sixteenth century embodied it in their lists.[5]

In those centuries when commercial communication was wholly by means of couriers who traveled regularly from city to city, a custom still determining the name of *correo* for a postman in certain parts of the world, the problem of the hare and hound naturally took on the form of, or perhaps paralleled, the one of the couriers. This problem was not, however, always one of pursuit, since the couriers might be traveling either in the same direction or in opposite directions.[6] This variant of the stock problem is purely Italian, for even the early German writers give it with reference to Italian towns.[7] As a matter of course also, it was varied by substituting ships for couriers,[8] while our modern text-book writers show their lack of originality by merely substituting automobiles for ships.

It was natural to expect that the problem should have a further variant, namely, the one in which the couriers should not start simultaneously. In this form it first appeared in print in Germany in 1483,[9] in Italy in 1484,[10] and in England in 1522.[11]

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[1] "De cursu canis ac fuga leporis."

[2] Folio 54; Unger, p. 106.

[3] "Una lepre e inanzi aun chane 3000 passi et ogni 5 passi delcane sono *p* 8 diquegli della lepre uosapere inquanti passi elcane ara giūto lalepre."

[4] "Vna lepre e dinançe a vn cane passa .60. e per ogni passa .5. che fa el cane la lèpre ne fa .7. e finalmente el cane lagiongni [la giongi in the edition of 1523, from la giúgnere, to overtake her] dimando in quanti passa el cane giōgera la lepre." Folio 42v. He says that the problem is not clear because we do not know whether the "passa .60." are leaps of the dog or of the hare, showing that he felt bound to take the stock problem as it stood without improving upon the phraseology. Indeed, we have few such marked examples of plagiarism, in that era of universal literary theft, as Pacioli's Sūma.

[5] Thus Rudolff (*Kunstlich rechnung*, 1526, 1534 edition, folio Nvj); Köbel (*Rechenbuch*, 1531, 1549 edition, folio 88, under the title "Von Wandern über Landt," with a picture in which the hare is quite as large as the hound); Cardan (*Arithmetica*, 1539, cap. 66); Wentzel (1599, p. 51); Ciacchi (*Regole generali d'Abbaco*, Firenze, 1675, p. 130); Coutereel (*Cyffer-Boek*, 1690 edition, p. 584), and many others.

[6] Various types are given in Pacioli's *Sūma* of 1494, folio 39.

[7] Thus Petzensteiner (1483, Folio 53), in his chapter "Von wandern," makes the couriers go to "rum" (Rome), thus: "Es sein zween gesellen die gand gen rum. Eyner get alle tag 6 meyl der ander geth an dem ersten tage 1 meyl an dem andern zwue etc. unde alle tag eyner meyl mer dan vor. Nu wildu wissen in wievil tagen eyner als vil hat gangen als der ander." Günther, *Geschichte*, p. 304; Müller, *Deutsche Blätter*, VI, p. 88.

[8] Thus Calandri (1491) says: "Una naue ua da Pisa a Genoua in 5 di: unaltra naue uiene dageno ua a pisa in 3 di. uo sapere partendosi in nun medesimo tempo quella da Pisa per andare a Genoua et quella da Genoua *p* andare a pisa in quanti di siniscon terrano insieme."

[9] Petzensteiner's arithmetic, printed at Bamberg.

[10] Borghi's arithmetic.

[11] Tonstall's *De arte supputandi*, folio 4, "Cvrsor ab Eboraco Londinvm proficiscens," etc. See also Cardan (*Arithmetica*, 1539, cap. 66, with various types); Ghaligai (1521, 1552 edi-

The invention of clocks with minute hands as well as hour hands gave the next variant, as to when both hands would be together—a relatively modern form of the question, as is also the astronomical problem of the occurrence of the new moon. The latest form, however, has to do with the practical question of a railway timetable, but here graphic methods naturally take the place of analysis so that of all the variants those of the couriers and the clock hands seem to be the only ones that will survive. Neither is valuable *per se*, but each is interesting, each is real within the range of easy imagination, and each involves a valuable mathematical principle—a fairly refined idea of function, and so it is probable that each will persist in spite of the present transitory period of the attempted debasement of elementary mathematics.

AN INVERSION OF THE COMPLETE QUADRILATERAL.

By J. W. CLAWSON, Ursinus College.

It is the purpose of this paper to point out an interesting example of the method of inversion. If a complete quadrilateral with some of its related lines and circles be inverted with respect to the quadrilateral's Wallace point, a new complete quadrilateral with some of its related circles and lines results. It is remarkable that this new quadrilateral is inversely similar to the original one, as will appear from II below.

Four straight lines (Fig. 1) ARB, BCP, CQA, PQR form a complete quadrilateral with A, P; B, Q; C, R for opposite vertices. The lines taken three by three also determine four triangles ABC, AQR, BRP, CPQ. It is well known that the circles circumscribing* these triangles are concurrent at a point O, the *Wallace point* of the quadrilateral,[1] that the circumcenters of the triangles are concyclic on l, the *circumcentric circle*,[2] that the orthocenters of the triangles are

tion, folio 64); Albert (1540, 1561 edition, folio Pi); Baker (1568, 1580 edition, folio 36); Coutereel (1631 edition, p. 371, and Eversdyck edition of 1658, p. 403); Trenchant (1566, 1578 edition, p. 280); Wentsell (1599, p. 51); Peletier (1549, 1607 edition, p. 290), Vander Schuere (1600, folio 179); Schonerus (notes on Ramus, 1586 edition, p. 174), and many others. Köbel's rather quaint German is interesting: "Zwen Burger vsz Oppenheym / einer Sō Heynrich / der ander Contz vō Treber gnant / wolten mit einander gen Rom gehn / vn̄ Heinrich was alt / vn̄ mocht einn tag nit mehr dan̄ zehen meiln gehn / Aber Contz vō Treber was jung vnnd starck / der mocht einen tag 13. meilen gehn / Deszhalben gieng Son Heynrich neun tag eh ausz Oppenheym dann Contz von Treber / Also war Son Heynrich Contzen 90. meilen furgangen / eh Contz angehaben hat auszugehn.

"Nun ist die frag / inn wie vil tagen Contz von Treber / Son Heynrichen übergangen / vnnd die zwen zusamen kommen seind." See also his *Zwey rechenbüchlin*, Frankfort, 1537 edition, folio 84.

* This figure is slightly distorted. The circles should join exactly through the points R and Q.

[1] "Scoticus," Leybourn's *Mathematical Repository*, 1804, Vol. I, p. 170. Mackay, *Proc. Edin. Math. Soc.*, Vol. IX.

[2] Davies, *Math. Repos.*, 1835, Vol. VI, Question 555 answered.

[1, 2] Steiner, Gergonne's *Annales*, 1828, Vol. XVIII, pp. 302, 303, 1°, 2°, 3°, 4°.